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PROJECT VASSEL XVI

COMPARISON OF PREDICTED AND MEASURED PROPAGATION LOSS VS RANGE

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DEPARTMENT OF THE NAVY
COMMANDER ANTISUBMARINE WARFARE FORCE, PACIFIC
FPO SAN FRANCISCO 96610

IN REPLY REFER TO
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Ser: 7/ 911
2 October 1969

From: Commander Antisubmarine Warfare Force, Pacific
To: Chief of Naval Operations
Commander Antisubmarine Warfare Force, Atlantic
Via: Commander in Chief, U. S. Pacific Fleet

Subj: ASW Fixed Wing Aircraft Evaluation Project (U)

Encl: (1) Project VASSEL XVI Report - Comparison of Predicted and Measured Propagation Loss vs Range

1. (U) Enclosure (1) is concerned with the ASW fixed wing project as tasked by CNO and is included in COMASWFORPAC's VASSEL (Validation of ASW Sub-Systems Effectiveness Levels) series. The report is being forwarded to COMASWFORLANT, acting for CINCLANTFLT, for inclusion in the FIXWEX evaluation project as appropriate.

2. (U) The analysis reported in enclosure (1) develops a methodology for comparing propagation loss predictions with loss measurements. This technique is then applied to loss measurements obtained during the VASSEL XV exercise and predictions made by Fleet Numerical Weather Central. Typically, measured losses and predictions have been compared by visually observing the correspondence of loss versus range plots. When, however, repeated measurements are made, as in VASSEL XV, information becomes available on the variability of the measurements. Wide variation in the measured data necessarily makes visual interpretation more difficult and less certain, thus, the statistical error analysis techniques applied in enclosure (1) provide more valid assessment of the prediction accuracy.

Harold G. Bowen

HAROLD G. BOWEN, Jr.

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I. INTRODUCTION

Mathematical models which accurately predict underwater sound propagation loss fields surrounding a given sound source would be extremely valuable to the Navy. Several such models have been developed, but their accuracy has not been established. Project VASSEL XVI intends to compare the losses predicted by a number of models to those observed in at-sea experiments.

The ultimate goal of the project is not the selection of one superior model; rather it is to investigate the usefulness of several models, each of which may be applicable for different purposes under various conditions.

This report is the first of the VASSEL XVI series and presents an analytic comparison between propagation loss data obtained during the VASSEL XV/FIXWEX exercise and losses predicted by a model developed and used by the Fleet Numerical Weather Central (FLENUMWEACEN).

Regardless of the degree of correspondence found between the model and the data, the model will not be considered validated for general use. The propagation loss measurements were made under controlled conditions but the geographical area and environmental conditions were extremely limited throughout the experiment. No information was obtained as to the applicability of the model under any other circumstance or in other areas.

This report is intended to:

1. Provide FLENUMWEACEN with a quantitative assessment of model performance against a set of high quality experimental data.

2. Provide information on model effectiveness to potential users of propagation loss predictions.

3. Indicate to those involved in designing at-sea experiments the nature of some unresolved problems which might warrant investigation.

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ABSTRACT

This report develops a methodology for analytically comparing underwater sound propagation loss measurements with predictions made by a propagation loss model. The method is employed to compare measurements during an at sea experiment (VASSEL XV/FIXWEX) with predictions made by Fleet Numerical Weather Central's propagation loss model. Experimental error and model error components are separated and their distributions are analyzed. The influence of local bottom topography on the experimental data and model predictions is examined.

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ACKNOWLEDGMENT

This report encompasses the efforts of numerous people. The propagation loss data were derived from another ASWTORPAC report, VASSEL XV /FIXWEX, the propagation loss model was developed by personnel of Fleet Numerical Weather Central, Monterey, who also provided appropriate model run data for this report; the analysis was done and the report prepared by Mr. J. [illegible] FAIRCHILD, Tactical Analysis Group, ASWTORPAC; Mr. R. M. LESSER, also of the Tactical Analysis Group, collaborated in the formulation of the analysis and interpretation of experimental results; CDR P. D. ROMAN, Technical Director, Analysis Division ASWTORPAC contributed valuable ideas and suggestions.

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I. INTRODUCTION

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II. METHODOLOGY

A. GENERAL. The various processes of gathering data at sea, performing the acoustic processing, developing the propagation loss model, and comparing results were accomplished under several different projects; VASSEL XVI only involves the comparison. A complete explanation of the comparison requires first a description of how the data were obtained and some explanation of how the model operates even though this work was done by others and is not part of the present study. Consequently, this section is sub-divided to provide brief descriptions of associated work done previously and some of the work done during this project.

B. DESCRIPTION OF VASSEL XV/FIXWEX EXERCISE.

While a detailed description of the VASSEL XV exercise is presented in a separate report, a summary of the exercise is provided here as background.

VASSEL XV was a joint effort of NADC, COMASWFORPAC and LTV Research Center. It was conducted during the period 28 April to 1 May 1968 in an area southwest of Oahu, centered at approximately 20°N, 161°W. The exercise involved an array of eight sonobuoys, the submarine USS SARGO (SSN-583), a low frequency sound projector towed by the USS RADFORD (DD-446), and monitoring aircraft from VP-28. The geometry of the array and the participants' tracks are shown in Figure 1.

Five similar events were conducted; each consisting of one pass by RADFORD towing the projector, and two passes by SARGO. A "pass" consisted of a transit between buoys C and F (Figure 1). Every attempt was made to maintain constant geometry throughout the exercise, with one exception. In four of the five events the projector was towed at 300 feet depth and the submarine was at 400 feet depth. In the fifth event both submarine and projector were at 200 feet.

Acoustic data were recorded by VP aircraft using calibrated AN/SSQ-48 XN-2 sonobuoys and specially configured and calibrated Absolute Sound Pressure Level (ASPL) panels. All sonobuoy signals were recorded continuously on magnetic tape.

The source levels of the submarine and the projector were obtained in subsequent measurements using the schooner FIESTA and the standard Navy PQM-2 noise measuring set. The projector was found to be omnidirectional with a source level of 89 db// 1 μbar at 125 Hz.

Detailed navigation logs were kept by all participants during the experiment. Local time standard, WWVN, was recorded on the magnetic tapes simultaneously with the sonobuoy signals to enable correlation with source position.

C. ACOUSTIC DATA PROCESSING

Both submarine noise and projector signals were recorded in the aircraft. Propagation loss measurements, however, were taken only from the projector runs. This section briefly describes the processing of projector recordings, by which measures of propagation loss were obtained.

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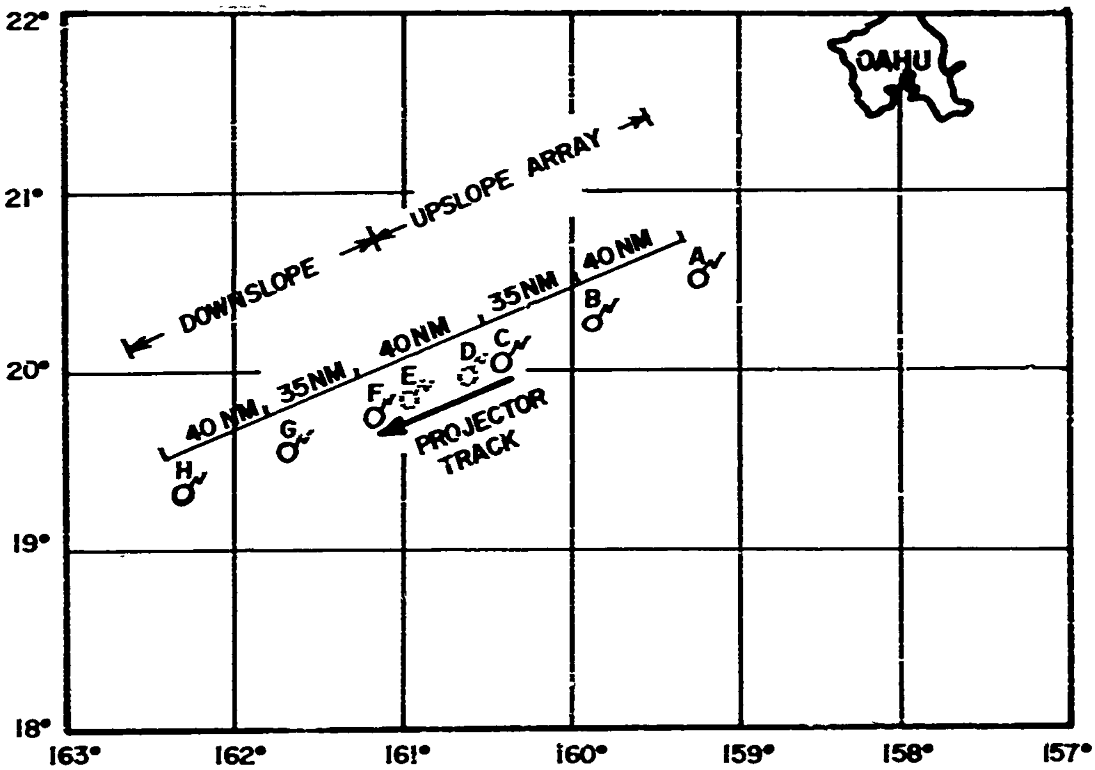


FIGURE 1

GEOMETRY OF VASSEL XV EXERCISE

The processing of recorded signals was done by the Ling-Temco-Vought Research Center as a part of the VASSEL XV project and a complete description of the equipment and methods used appears in that project report.

The basic objective of the recorded signal analysis was to obtain measurements of the propagation loss as a function of range from the source. The results of the analysis consist of two series of numbers, each number being the loss observed in a particular range interval from the source. In one series the numbers are one minute averages and in the other the numbers are five minute moving window type averages. Incident to determining propagation loss the signal to noise ratio in the signal band and the noise level in adjacent frequency bands were also obtained.

The processing procedure used both analog and digital techniques. The raw data tapes were time compressed, edited, filtered and re-recorded. The edited tapes contained one channel of signal plus noise, two channels of noise in frequency bands adjacent to 125 Hz and one channel of time markers and event identification. The signal band was 2 Hz wide and the noise bands were 12.5 Hz wide, centered at 110 Hz and 140 Hz. The output of the edited tapes was heterodyned to base band, detected and sampled by an 11 bit analog to digital converter. The resulting power values for signal plus noise and two noise channels were then recorded on digital tapes.

The digital records were read into an IBM 360/50 computer which performed a number of adjustments, such as:

1. Compensation for losses in analog processing.

2. Compensation for loss rate as a function of frequency (4.5 db/octave).

[illegible] Compensation for attenuation changes in receiver channels.

[illegible].ata were then transformed to 1 Hz reference levels and averaged [illegible] and five minute "moving window" periods. The signal to noise ratio in the signal band was calculated and finally the propagation loss was computed from:

$$\text{P.L.} = \text{Source level (89 db)} - \text{Signal level} \left[(S+N)_B - N_1 B \right]$$

where:

$(S+N)_B$ = Signal plus noise power in signal band B

N_1 = Noise power in 1 Hz band

B = Signal band width (1.25 Hz)

No calculations of propagation loss were made if the signal to noise ratio was less than 3 db.

The propagation losses were also averaged over one and five minute moving windows and final output tapes were recorded. They contain:

1. One and five minute noise averages at each sonobuoy.

2. Propagation loss about each buoy as a function of range and averaged for one and five minutes.

3. Signal to noise ratio as a function of range about each buoy. Only the five-minute averaged data were available for VASSEL XVI, so the remainder of the report deals with five minute averages only.

D. PROPAGATION LOSS MODEL

The propagation loss model now in use at FLENUMWEACEN is the most recent and most comprehensive of a series that have been developed by that command. The present version was put into general use in October 1968.

The basic objective of the model is to calculate the sound propagation loss at points in a field about a given source. It consists of a computer program which accepts certain inputs concerning the situation to be modeled, performs ray tracing and loss calculations, and tabulates the results as propagation loss vs. range from source.

For purposes of description, the propagation loss program may be conveniently broken into a number of sequential steps. The first step is to accept input data concerning the geometry of the situation and the characteristics of the source, the medium and the boundaries. Having this data the program then traces out the geometrical paths followed by approximately 100 representative rays emanating from the source. The angular spacing of rays traced varies around the source; the spacing normally used is shown in Table 1.

ANGLE INTERVAL	SPACING
0.0 - 14.75 (degrees)	0.25 (degrees)
15.0 - 39.00	1.00
40.0 - 56.00	2.00
58.0 - 88.00	5.00

0 degrees = horizontal ray
± 90 degrees = vertical ray

RAY ANGLE INTERVALS USED BY FLENUMWEACEN

TABLE 1

At any point on one of the rays, the propagation loss from the source to that point along the ray may be calculated as the result of absorption, spreading, and reflection losses. The program computes and tabulates these losses at intervals along each of the rays.

At this stage the program has traced rays and computed losses along these rays. This does not mean, however, that the total propagation loss between the source and all points in the field is known. For any point in the field about the source there are several paths by which sound may propagate from the source to the point. There is generally one direct path, one surface reflection, one bottom reflection and combinations of multiple surface and bottom reflections which connect the source and the point. The total loss between the source and the point is a combination of the losses along each of the various acoustic paths.

Most points picked at random in the loss field do not lie on any of the representative rays which were traced out, but lie between a pair of direct rays, a pair of surface reflected rays, a pair of bottom reflected rays, and so on. To determine the loss along an untraced direct ray, the program interpolates between losses found for points on the two adjacent direct rays. Interpolation is similarly used to get losses along the reflected rays. When the losses along the various paths have been determined a total loss for the given point is calculated. This total loss is less than the loss along any single path.

The general objective of the program is to tabulate the predicted loss at a receiver at a given depth as a function of horizontal range from the source. For example, if the loss at half-mile intervals along the 100 foot depth level is desired, the program selects points a half-mile apart at 100 foot depth and computes the total loss from the source to each point.

The above program is analagous to having a sound projector in a fixed location and measuring the loss to a receiver which is towed along at 100 feet. A minor problem arises when the program results are to be compared to measurements taken with a fixed receiver and a moving source. Initially, it appears that a complete new ray trace must be made for each position of the source as it is moved along. This difficulty is avoided, however, by reversing the roles of the source and receiver in the model. In other words, if the experiment requires a fixed receiver and a moving source, the computer program is run as if the source were in the receivers position and vice versa. This role reversal avoids having to run multiple ray traces for each new source position. Justification for this procedure is the widely used reciprocity relation which holds that, given a source and receiver at fixed points, an identical transmission loss will be observed between the two points if the source and receiver are interchanged.

When the model is to be used to predict losses for a specific area and time, it must receive certain information concerning the circumstances to be simulated. The model requires the following input data:

- Source and receiver depths.
- Source frequency.
- Sound velocity profiles (SVP's), surface to bottom.
- Bottom depth and topography.
- Bottom roughness and reflectivity.
- Ray spacing to be used.

Prior to the input of the SVP's, each is fitted to a cubic curve. The result is a smooth curve with no discontinuities in slope, thereby eliminating false caustics which would occur at discontinuity points in a series of linear segments. If the dimensions of the area of interest are large enough to include two or more differing water columns, an appropriate velocity profile is introduced for each, and linear interpolation between profiles is used to obtain a continuous

field. If BT drops have been made in the area to be simulated, the velocity profiles are computed using the BT information. Salinities are obtained from climatological charts. For depths beyond the BT data climatology is used to extend the profile to the sea floor.

Bottom depth and topography are obtained either from measurement during experiments or from the most accurate available charts of the area. Generally, the depth at each half-mile interval is specified to the model. Depth data used during the VASSEL XVI model runs are from charts of the area.

Bottom roughness classes are obtained from Marine Geophysical Survey Data where available. The sea floor is divided into five roughness classes and bottom losses as a function of the frequency are obtained for each class by analysis of the MGS data.

The program output consists of three parts:

1. A specification section which identifies the run, the source and receiver depths, and the frequency.

2. A tabulation of propagation loss at every horizontal half-mile range from 0 to 125 miles. The losses are shown to the nearest .1 db.

3. A graphic plot of the tabulated data showing the losses to the nearest 1 db every one nautical mile from 0 to 125 miles.

The computer time used for elements of the program is as follows:

Ray Trace - 3.5 hours per run (approximately)

Propagation Loss - .2 hours per run (approximately)

E. GEOMETRY OF EXPERIMENT AND MODEL

The at-sea experiment was conducted with a particular geometrical relationship between the buoys, the moving source, and the bottom terrain. It was important that the model runs be based on the same geometry. In setting up the model, the first step, as explained in Section D, was to reverse the roles of the source and receiver, since the source was the moving element. Thus, the program regarded each buoy as a source and predicted the propagation loss that should be observed by a receiver towed along the projector's track.

Figure 2 is a schematic vertical section of the experiment. The diagram is out of scale to emphasize the influence of the irregular bottom on the sound propagation paths. The buoys are designated A thru H. The line O-P represents the path over which the projector was towed. Point O, directly below buoy C, and point P, directly below buoy F, are 40 miles apart.

Examination of the signal paths to each buoy revealed that signals arriving at buoy A originated 75 to 115 miles away, signals at buoy B originated 35 to 75 miles away and signals received at buoy C originated 0 to 40 miles away. Data from buoys A, B, and C were therefore three independent, contiguous sets that covered an overall range of 0 to 115 miles with one overlap in the 35 to 40 mile range. Similar, but not identical,

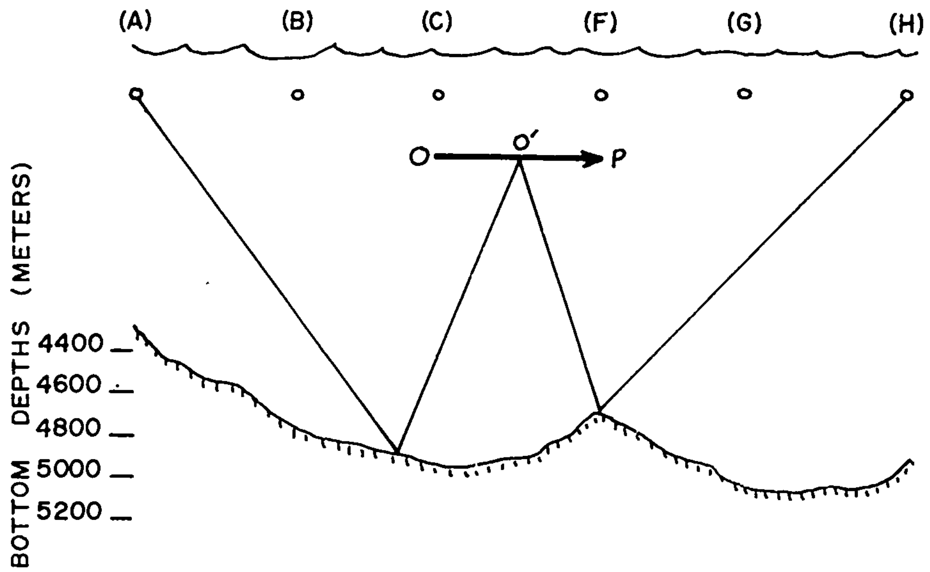


FIGURE 2
VERTICAL SECTION OF
EXPERIMENT AREA

results were obtained for buoys F, G and H.

It is important to keep the results from buoys A and H separate, even though they measure losses over the same range interval. Since the bottom terrain was quite different, about A and H, the measured losses were not expected to be identical. The same reasoning was applied to buoy pairs B - G and C - F. Thus, data from buoys A, B, and C show losses over the 0 to 115 mile range interval up slope from the source, and F, G, H show down slope losses over the 0 to 115 mile interval. The data gathered at buoys D and E were not used in this report due to lack of computer time to run ray traces and the over-lap with data from buoys C and F. Finally, the measurement data consisted of six independent sets, each set covering a particular 40 mile range interval from a particular buoy. There were two sets of measurements for each interval, but they were not combinable.

The above considerations dictated what model runs were to be made. First, of course, it is not correct to make one ray trace and produce one loss curve to represent the entire area. Rather, a trace was made about each buoy and the appropriate 40 mile segment of the model loss curve was used for comparison with the measured results.

F. COMPARISON METHODOLOGY

One of the primary objects of this project is to investigate the degree of agreement between model predictions and observed propagation loss measurements. Experimental results show, however, that measurements repeated under similar conditions vary significantly and do not produce any unique data set that can be considered the "true" loss. Thus, at any particular range several values exist for measured loss and the first problem was deciding which the model should agree with. Having decided what the model should agree with, it was necessary to decide under what conditions agreement did or did not exist. Then, the final step was to separate the differences between the model and the observed data into meaningful components. It is in doing this that we hope to derive the most significant insights into the problem.

At this point, it will be helpful to discuss some broad aspects of the process of taking physical measurements. The ideas are quite elementary, but they have a crucial bearing on how the comparison should be made and on what conclusions we may draw.

First, as a matter of definition, both the model and the measurement process involve several "controlled" and "uncontrolled" variables. In the model context, the controlled variables are those for which specific values are input to the model. These include: the source - receiver geometry; the sound velocity profile; the bottom topography; the bottom roughness and reflection coefficients. The uncontrolled variables are those for which some average or random value is assumed, or those which are disregarded entirely. These include: the phase relationship of multipath arrivals; scattering due to small objects such as fish; variation of the water mass due to currents and other transitory effects.

In the experimental context, the controlled variables are those which are held constant or permitted only limited variation. Ideally, controlled variables are constant and measureable throughout an experiment. In this experiment the source - receiver geometry, the sound velocity profile, and the shape and physical characteristics of the boundaries are all considered controlled variables. The uncontrolled variables are similar to those assumed for the model.

When measurements are made under "constant conditions" this means only that the controlled variables should remain constant. The uncontrolled variables are permitted to vary as they will. We note, however, that during any particular single measurement the uncontrolled variables assume a single set of particular values, which may or may not represent their average value. During the next measurement they assume a new set. In a well conducted experiment, it is only the variation of the uncontrolled variables which produces discrepancies between one measurement and the next, if they are made under "constant conditions".

With respect to the model, "constant conditions" means that the controlled variables remain fixed and the model will produce an identical result time after time if average values are used for the uncontrolled variables. If randomness is introduced in the uncontrolled variables, then the results will vary from run to run even under "constant conditions". The FNWC model uses average values for uncontrolled variables, so for one set of conditions it will always produce an identical prediction.

Next we look briefly at the notion of a "true mean loss'. If we visualize a lengthy experiment which is conducted under constant conditions, we expect the uncontrolled variables to fluctuate many times throughout their possible ranges. If numerous measurements are made during this time and the results averaged, the average will approach some hypothetical "true mean loss" for that particular set of controlled conditions. Accurate knowledge of this quantity would be of considerable value. The difficulty, of course is that this process uses an inordinate amount of time and creates great difficulty in keeping the controlled variables constant. So this "true mean loss" is a quantity which surely exists, but which we do not measure. In experiments such as VASSEL XV, we compromise and take a few measurements with which we estimate what would happen if we took many. The crucial point, of course is that four measurements do not produce a "true mean" but rather an imperfect estimate of it.

Returning to the problem of what the model should represent, we have a clear solution. The model should be a replica of the "true mean loss" for the particular set of controlled conditions involved.

It is important to recognize here that even if we have a perfect model, in that it does represent the "true mean loss" at every point, we do not expect the measured data to agree with it perfectly. When we take a few measurements, the average of the set is expected to differ from the true mean. We cannot, therefore, simply observe that the model differs from the average of these few measurements and thereby conclude that the model is deficient. Rather, it is necessary to analyze the relationship of both the model and the data to the hypothetical true mean, as well as their relationship to each other.

We will use the term "error" hereafter to refer to the differences between model or measured values and the true mean. Use of the word "error" does not imply mistakes by the experimenters or model builders. The word simply means that a difference exists between two quantities. Thus, for the difference between the model and the true mean we use the term "model error" and for the differences between measurements and the true mean we use "experimental error" or "sampling error". The differences between the model and the measured data are not properly referred to as error but as model-data discrepancies. Model-data discrepancies contain elements of both model error and experimental error.

We now turn to the method adopted to calculate the mean propagation loss from a number of measurements. The term "mean propagation loss" expressed in db does not, by itself, specify whether the 10 log operation precedes or follows the averaging operation. Thus, it may refer either to 10 log E $[x_i]$ or E [10 log x_i], where E [ ] represents the expectation of the function in brackets, and x_i is one propagation loss measurement expressed as an intensity loss ratio. A decision as to which is the appropriate method of averaging a set of loss measurements depends partially upon the use to which the results will be put; neither method is always right nor always wrong. The former method, i.e. taking the logarithm of the average loss, is appropriate to specify the output of a system which responds linearly to energy input. The latter method, i.e. averaging the logarithm of the measurements, or db, is appropriate for specifying the average output of a system which responds logarithmically to energy input. Two examples of the latter would be devices which measure db and the human ear. Because there are so many more devices which respond linearly than logarithmically to energy input, and because of the generally greater usefulness of the concept of average power than the average logarithm of power, the former method is appropriate for the great majority of purposes.

By the above criterion, the appropriate method for averaging measured losses in this report would be to average the loss ratios and transform the average loss to db. There are, however, over-riding considerations which make it desireable to adopt the other method, i.e. averaging the db.

We do not contend that the sample means obtained this way are identical to average intensity losses, but rather that they are more useful for our purposes.

Much of the work done for this report involves investigation and use of the statistical distribution of propagation loss measurements, and the application of certain statistical tests. The majority of these statistical procedures require that one deals with an approximately normal (Gaussian) population. It was found (see Section III-C) that the distribution of losses, stated in db, is very close to normal, and the distribution of loss ratios is log-normal. Since the log-normal distribution is very difficult to work with statistically we have adopted the procedure of averaging and analyzing the distribution of the db measurements which are normal. A number of other factors justify using this averaging method.

1. The major conclusions of the report will be the same no matter which method is adopted. Some of the numerical results will differ, but the difference is analogous to reporting weights in grams rather than pounds. One may convert back and forth.

2. The difference between the "true mean losses" calculated by each method is both small and calcuable. For the data used in this report the average difference can be shown to be 1.4 db. Thus, if the model is found to be a perfect predictor of true mean db, it will differ by 1.4 db from the true mean intensity loss, on the average.

3. A description of the distribution of db is just as useful as a description of the distribution of intensity ratios. Given either one we may calculate the other, as well as related quantities.

G. DEFINITIONS

Several terms are used in the remainder of the report which have commonly known meanings, but which are used very specifically here. It is important to understand the restrictive meaning given them here.

1. "Measurement". This term is used to denote the smallest discrete amount of data used. As the projector is towed toward or away from a buoy, a continuous range-loss curve is generated. This curve is then broken into one mile segments and the average loss in each segment is termed a "measurement". Towing the projector for 40 miles thus produces 40 "measurements" at each buoy. Since six buoys were used and there were four projector runs, we have 40 x 6 x 4 = 960 possible measurements. Due to some missing data only 674 of these were actually obtained.

2. "Sample". A sample is generally made by grouping measurements into sets of four. Each sample thus contains four (or less) repeated measurements made under the same conditions. The four measurements made while the projector was 25 miles from buoy "C" are one "sample". Some samples contain less than four measurements due to missing data.

3. "Sample Mean". If the measurements in one sample are averaged the result is the "sample mean". It is the average loss, in db, for that sample.

4. "Sample variance" is the variance of measurements in a sample.

5. "Pooled variance" is a weighted average variance for several, or all, samples combined. The weighting takes into account the varying number of measurements in each sample.

III. RESULTS AND DISCUSSION

A. GENERAL. The general, overall results of the VASSEL XV measurements are depicted in Figures 3 and 4. Up slope and down slope measurements are separated for reasons discussed in Section II. The figure shows one loss trace for each projector run, one trace which is the average of losses from the individual runs and one trace which shows the loss predicted by the model. This figure contains several important features which are analyzed in detail in subsequent sections.

1. At any particular range, considerable variation exists in the measurements from one run to another. Thus, repeated measurements made between the same points and under similar conditions do not yield consistent results. Frequently differences of over 10 db can be found in repeated measurements.

2. The trace for each individual pass represents the value that would be found in a five minute, moving window, averaging device. This has a very definite smoothing effect on the data, but short term oscillations of over 10 db in a small number of miles are nevertheless quite common. The individual measurements generally oscillate more widely than the average.

3. The model generally predicts less loss than the measurements indicate. Measurements and predictions agree better at short range, perhaps to 30 or 40 miles, than at long range where the model consistently predicts too little loss by several db.

4. In regions where the measurements show a definite repeatable "fine structure", i.e. oscillation patterns which are consistent from run to run, the model seldom follows the pattern. In general, the model is much smoother than either the individual runs or the four-run average particularly at longer ranges, where the model often remains in a 1 or 2 db interval for many miles.

5. Each individual run displays oscillations which do not appear to vary conspicuously with range; that is, the oscillations observed in the data at ranges from 100 to 115 miles are not significantly different than those in the 30 to 50 mile interval. Also, the oscillations are apparently random in the sense that they are not consistently repeatable from run to run. With few exceptions, the patterns observed in one run are reminiscent of those seen in other runs but cannot be superimposed to produce a consistent pattern.

6. There is further evidence of randomness in the individual runs, in that they do not follow simple patterns expected of interference between plane waves. The loss curves do not have consistent oscillatory periods nor amplitudes and do not display the arch-like patterns characteristic of interference of simple multipath propagation modes.

B. MODEL-DATA DISCREPANCIES. The previous section presented a general graphical picture of how the model predictions and the measurements agreed. In this section a quantization of the model-data differences is presented which is intended primarily to be useful to potential users of the model. We do not yet address the probabilistic problems involved in comparing the model to a hypothetical "true mean loss". Rather, we use the directly observable

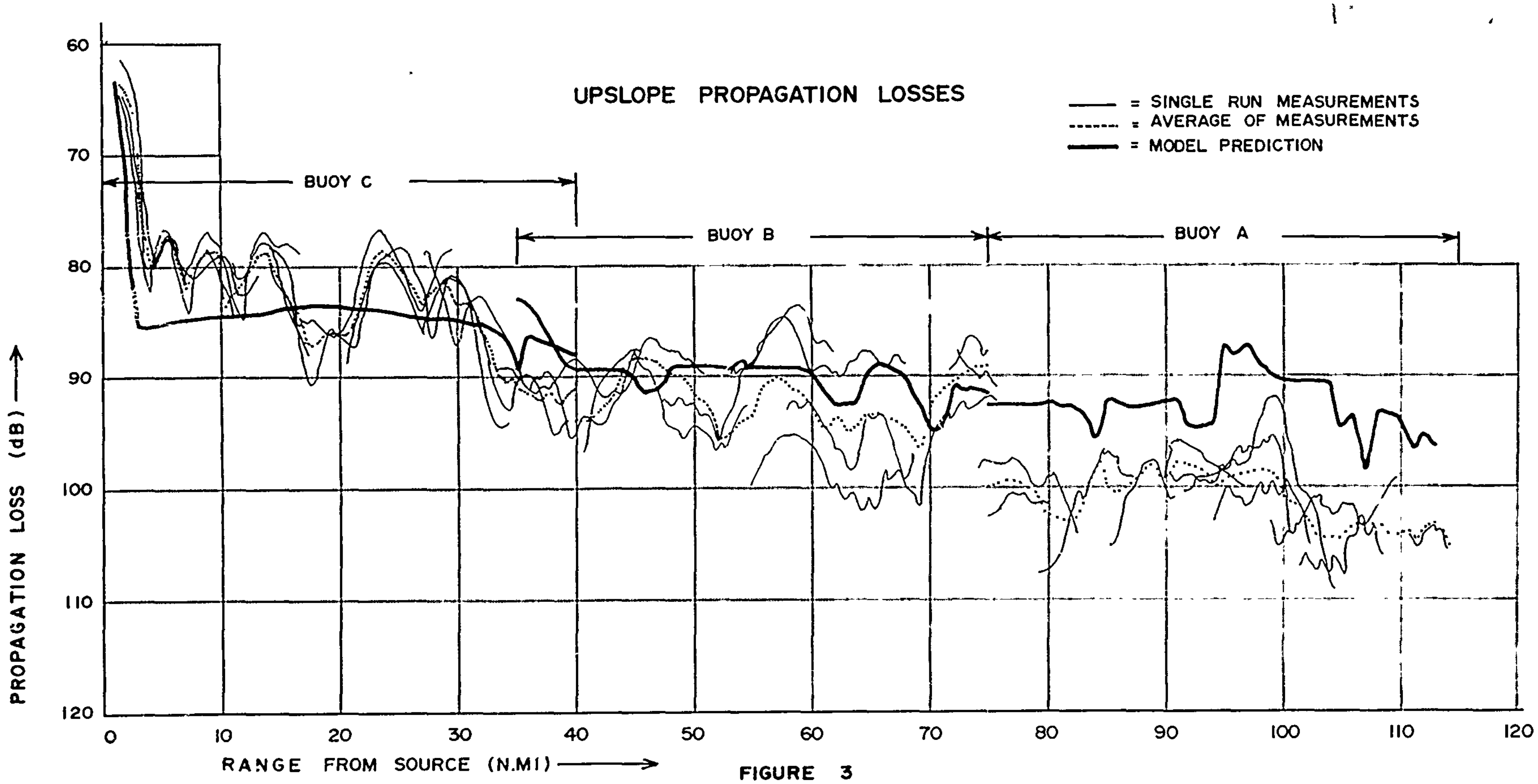

UPSLOPE PROPAGATION LOSSES
= SINGLE RUN MEASUREMENTS
= AVERAGE OF MEASUREMENTS
= MODEL PREDICTION
BUOY C
BUOY B
BUOY A
60
70
80
90
100
110
120
PROPAGATION LOSS (dB)
0
10
20
30
40
50
60
70
80
90
100
110
120
RANGE FROM SOURCE (N.MI)

FIGURE 3

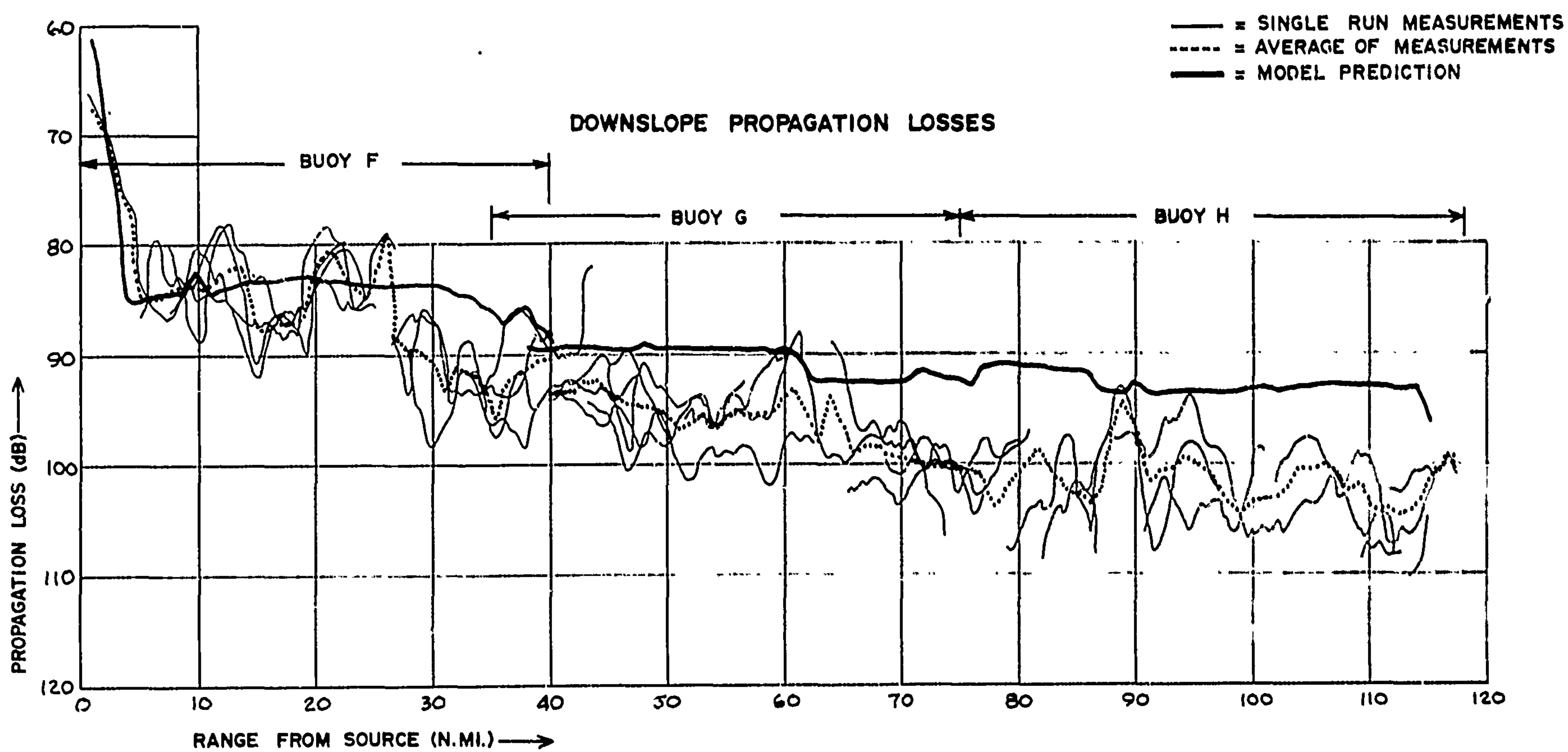

= SINGLE RUN MEASUREMENTS
= AVERAGE OF MEASUREMENTS
= MODEL PREDICTION
DOWNSLOPE PROPAGATION LOSSES
BUOY F
BUOY G
BUOY H
PROPAGATION LOSS (dB)
60
70
80
90
100
110
120
0
10
20
30
40
50
60
70
80
90
100
110
120
RANGE FROM SOURCE (N.MI.)

FIGURE 4

model-data discrepancies to indicate to potential model users what agreement might be expected between the model and losses which are measured or experienced. Even the user who makes no measurements as such, but only listens to a sonobuoy, is interested in this relationship, for he is really experiencing one unique loss value which is different than the true mean.

The data is organized into samples which contain several measurements and one model prediction. The differences between each measurement and the corresponding prediction are plotted in Figure 5. The differences are plotted separately for each buoy. We see here, for example, that eight of the measurements obtained at buoy C exhibit between 7 and 8 db less loss than the model predicted. Also shown on each plot is the mean or average difference between all the measurements made at a buoy and the model prediction. For example, the measurements made at buoy B averaged about 2.8 db more loss than the model predicted. Some pertinent features shown here again:

- Agreement between the model and the data is better at close range than at long range.

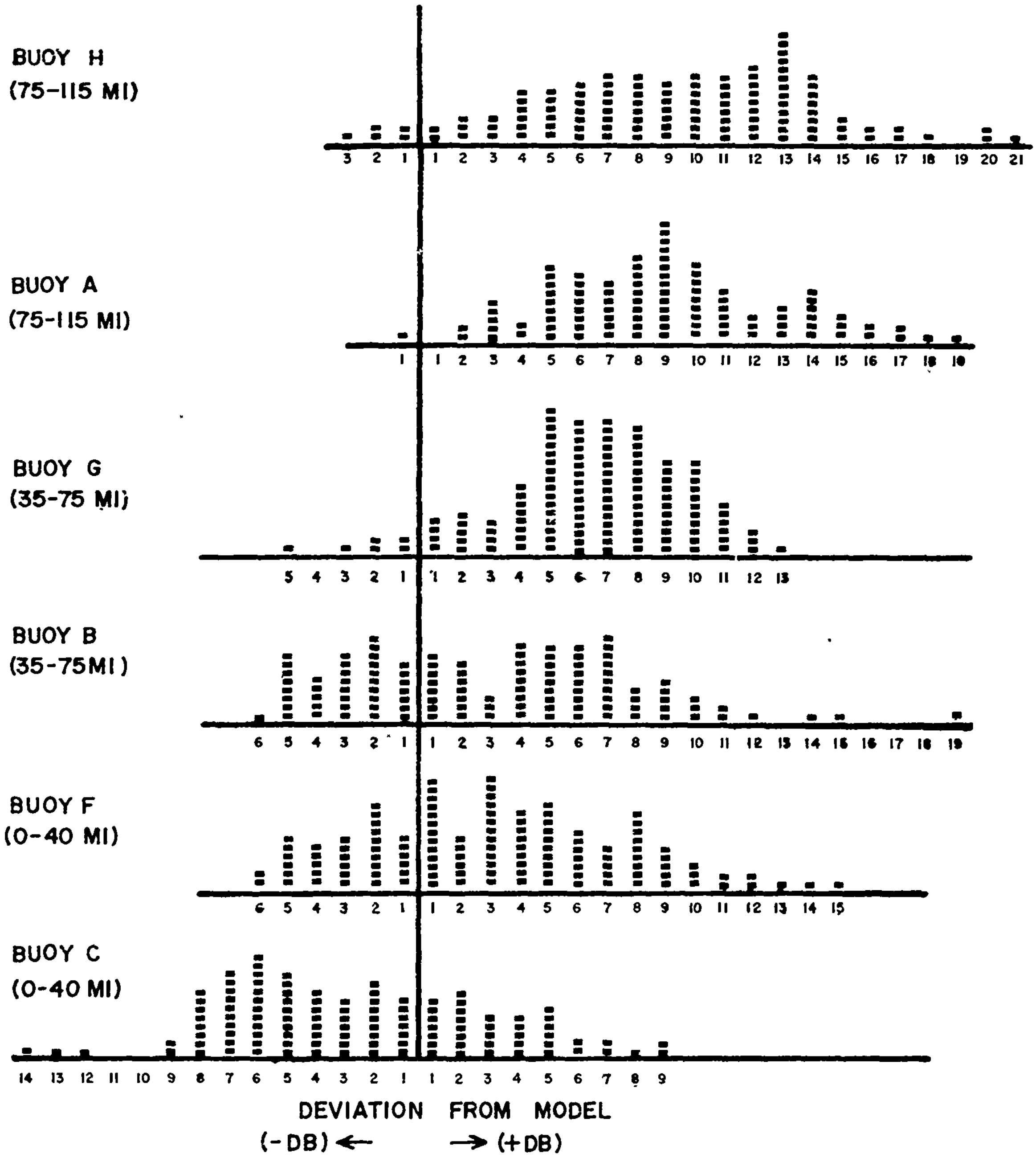
- The measurements are quite widely scattered about the model. Generally about 20 db separates the deviation limits at each buoy.

Rather than providing a basis for evaluating the model, the information contained in the figure indicates to a potential user of the model what he might expect in the way of accuracy. The figure says that the model does agree fairly well on the average at close range, but that any individual measurement is likely to deviate from the model by several db. At longer ranges, the model predicts consistently too little loss for the particular area where those measurements were made. To see how the model agreed over all — for all ranges — the data in Figure 5 is summed into one plot in Figure 6. This figure shows the relation of all measurements to the model. The average deviation is about + 4.5 db. This does not mean the average model error is 4.5 db, but only that on the average the measured losses were 4.5 db more than the model predicted.

Figure 7 presents the cumulative distribution of the data in Figure 6. It shows what fraction of all the measurements are within so many db of the model. For example, 50% of the measurements are within 5.5 db of the model, 80% are within 9.0 db and 95% are within 13 db.

C. EXPERIMENTAL ERROR ANALYSIS.

1. General. As noted earlier, the measurements show considerable variability within each sample. In other words, repeated measurements made between the same two points, under nominally identical controlled conditions, differ significantly from one to another. These differences are "experimental error". It is important to examine the experimental error, for it bears heavily on what sort of conclusions may be drawn from the data and it has strong implications concerning future propagation loss measurement experiments. Some of the factors involved are:



MEASUREMENT DEVIATIONS FROM MODEL

FIGURE 5

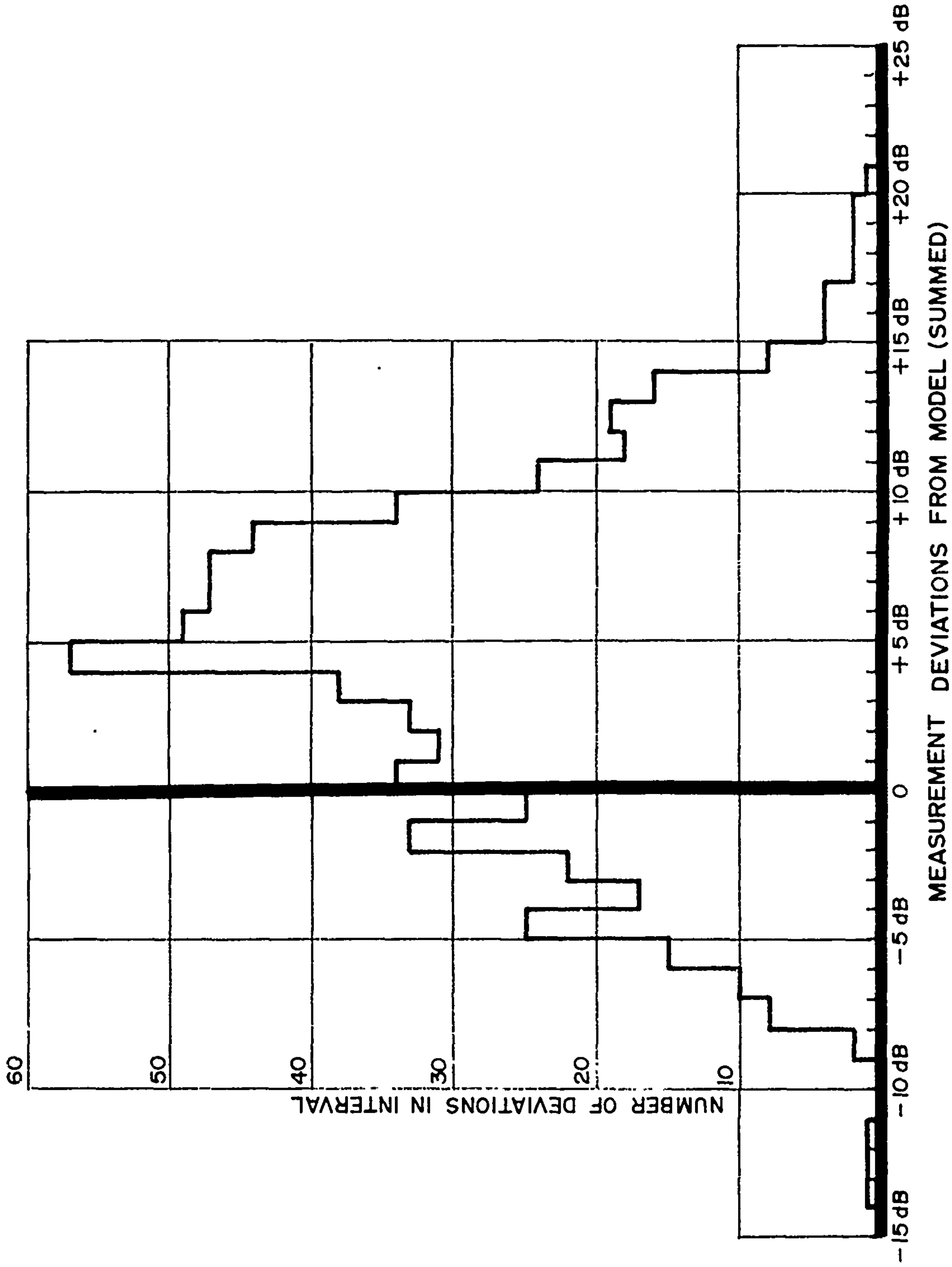

NUMBER OF DEVIATIONS IN INTERVAL
60
50
40
30
20
10
-15 dB
-10 dB
-5 dB
0
+5 dB
+10 dB
+15 dB
+20 dB
+25 dB
MEASUREMENT DEVIATIONS FROM MODEL (SUMMED)

FIGURE 6

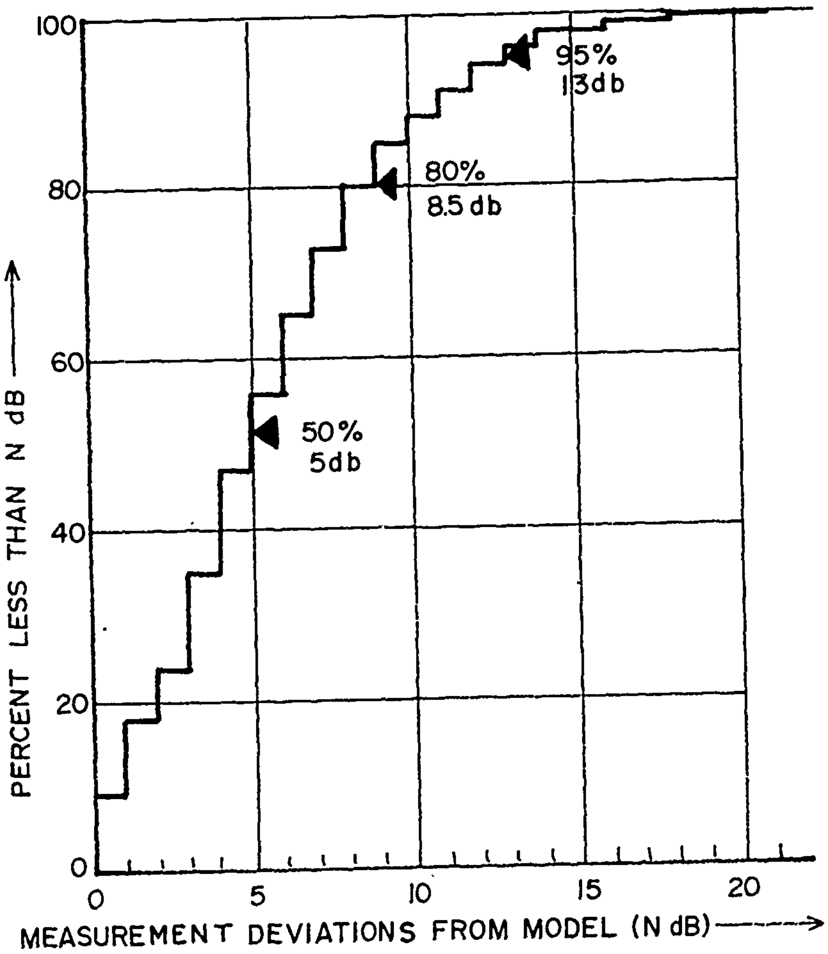


FIGURE 7

CUMULATIVE DISTRIBUTION OF MEASUREMENT DEVIATIONS FROM MODEL

• Generally, as the experimental error in a set of measurements increases, so also does the number of measurements required to obtain an acceptable estimate of the true mean. Conversely, the confidence that any single measurement is close to the mean decreases.

• If a set of measurements has a large experimental error, we can expect the means of samples to differ considerably from the true mean. If we try to make comparisons between a theoretical model and some measurement data, we are less certain whether the observed differences are due to model error or experimental error.

• The value of a propagation loss model is not determined solely by how well it predicts the mean propagation loss between two points. A given model could be an acceptable, even perfect, predictor of true mean loss, but fail to be useful because actual loss values are widely distributed about the true mean. In other words, a precise prediction of a true mean may not tell much about what will result from a single measurement if the experimental error is significant.

• If data variance, or experimental error, for a particular set of controlled conditions are found to be too high, it can be reduced only by "controlling" one or more of the now uncontrolled variables. In the model context, this means using specific rather than average or random values for a now uncontrolled variable. Investigation of the data variance may thus encourage or discourage further model development.

• Referring to plots of mean propagation loss vs range and model loss versus range, in Figures 3 and 4 we see that the loss measurements are much more oscillatory with range than the model predicts. A serious question arises as to whether the data oscillation is due primarily to changes in range and the associated controlled variables or due to random fluctuation of the uncontrolled variables. If the latter is the primary cause, the true mean loss might be a fairly smooth curve, much like the model. This does not suggest that oscillations do not really occur in individual measurements — they do. Rather, the question addressed is the character of the oscillations. Investigation of the experimental error can produce some insight into this question.

2. Sample Variance. The basic measure of experimental error is given by the "sample variance" which is defined by the relation:

$$S_i^2 = \sum_j \frac{(\overline{X}_i - X_{ij})^2}{N_i - 1}$$

where S_i^2 = variance of the $i^{\underline{th}}$ sample

$\overline{X}_i$ = mean of the propagation loss measurements, in db, in sample i

X_{ij} = the $j^{\underline{th}}$ measurement of propagation loss in the $i^{\underline{th}}$ sample

N_i = number of measurements included in sample i

It is appropriate, also, to introduce the notion of a "sample space". This term simply refers to the set of conditions that prevailed for the i^{th} sample. Or, it is the particular state of the controlled variables which prevailed for the i^{th} sample. Our sample spaces are defined by a one mile range increment referenced to a particular buoy, a particular velocity profile, a particular bottom contour and so on.

The sample variance was calculated for each of the 71 samples which contained 4 measurements and the 89 samples which contained 3 measurements. A tabulation of the variances, along with other data arranged by sample is shown in Appendix A. In order to progress toward a general description of experimental error, we form the distribution of sample variance as shown in Figure 8. This figure shows, for example that four samples had a variance of between 22 and 24 db^2, and 24 samples had a variance between 4 and 6 db^2. It should be recalled that these are values for the variance, not the standard deviation or familiar "one sigma". The "sigma" is the square root of these variance figures.

3. Population Variance. Figure 8 shows that sample variance ranges from less than 2 db^2 for 27 samples to over 90 db^2 for one dispersed sample. This disparity of variance is addressed below.

If, as in developing the idea of a "true mean", we regard the propagation loss at a single sample space as a random variable, it will have some "true variance" as well as a "true mean". Again, it could be measured only over a time period long enough to include a number of oscillations of the uncontrolled variables. This hypothetical true variance is called "population variance".

The observed sample variances are subject to the same vagaries of the sampling process as are the sample means. Thus, individually they are imperfect estimates of the population variance.

A description of the population variance involves determining how variance changes from one sample space to another. So before attempting this. we test to see if it can be shown that population variance changes at all. If we cannot show that it does, in fact, change, obviously we cannot describe how it changes. The best description of the population variance in that event is that it is constant and is equal to the pooled variance of all samples.

The method used to do this is statistical hypothesis testing. So we advance the hypothesis the population variance is constant in all sample spaces and is equal to the pooled variance of all samples. In terms of the experiment, we test the hypothesis that the true variance of possible measurements is the same at one point as at any other in the experiment area and does not change significantly with range from a buoy. The observed differences in sample variance are attributable to the sampling process, if the hypothesis is accepted.

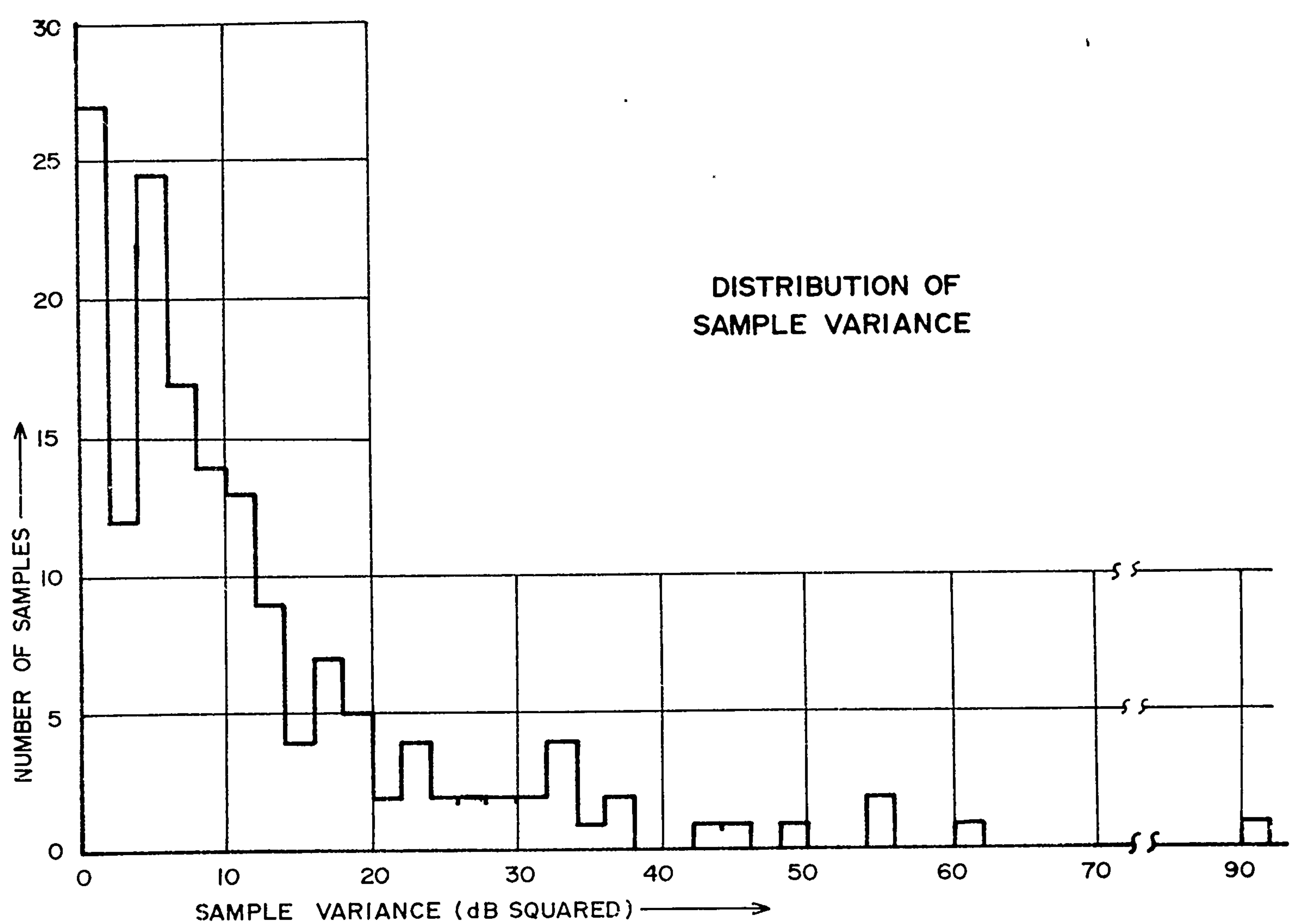

DISTRIBUTION OF
SAMPLE VARIANCE
NUMBER OF SAMPLES
30
25
20
15
10
5
0
10
20
30
40
50
60
70
90
SAMPLE VARIANCE (dB SQUARED)

FIGURE 8

The "pooled variance" is a combination of all the sample variances with weighting by sample size. It is calculated from the relation:

$$S_p^2 = \frac{\sum (n_i - 1) S_i^2}{\sum (n_i) - k}$$

where:

n_i = number of measurements in the i^{th} sample

S_i^2 = sample variance of the i^{th} sample

K = number of samples used

For the 160 samples which contained either 3 or 4 measurements, the pooled variance was:

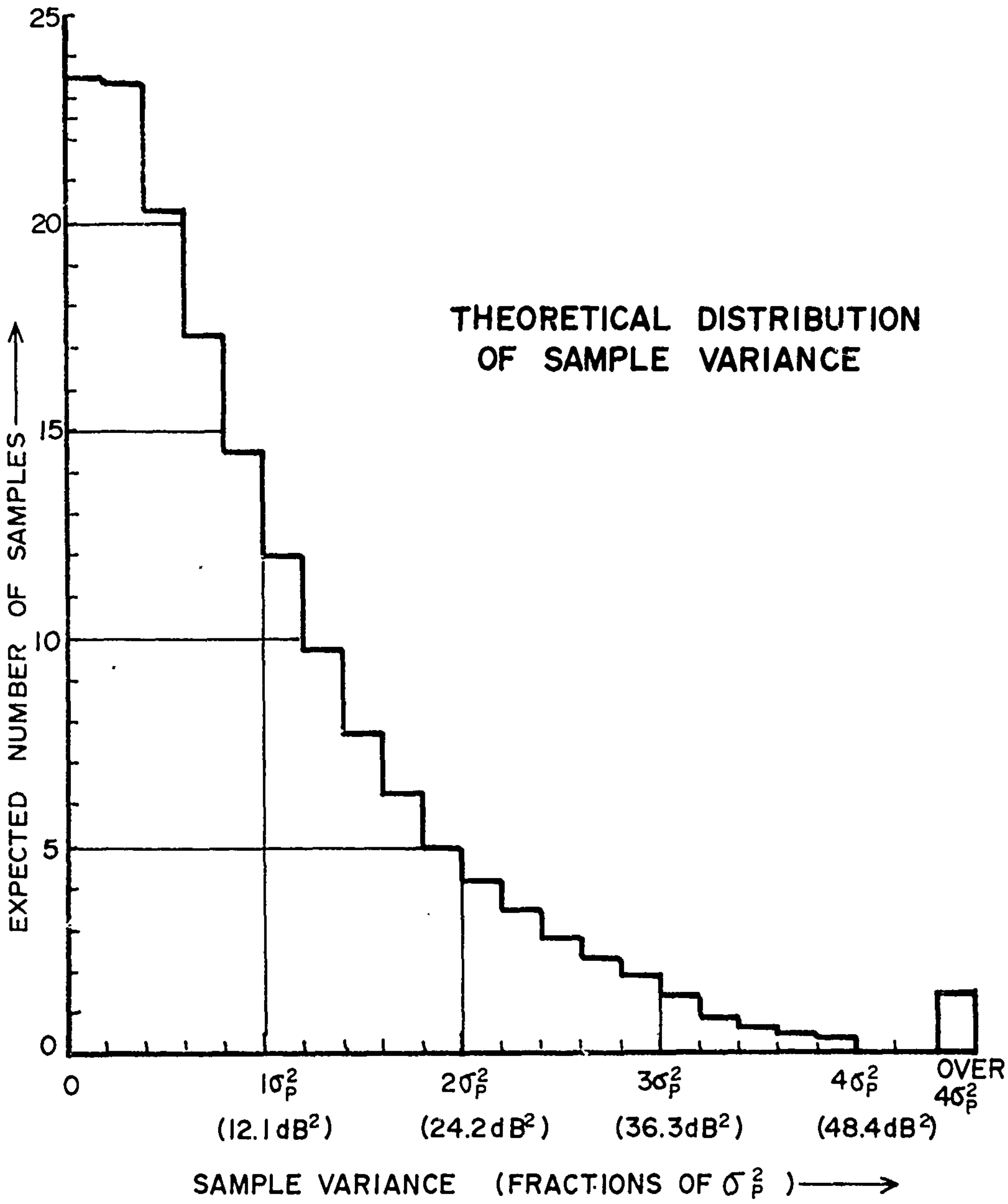
$$Sp^2 = 12.1 \text{ db}^2$$

And the pooled standard deviation was:

$$Sp = 3.48 \text{ db}$$

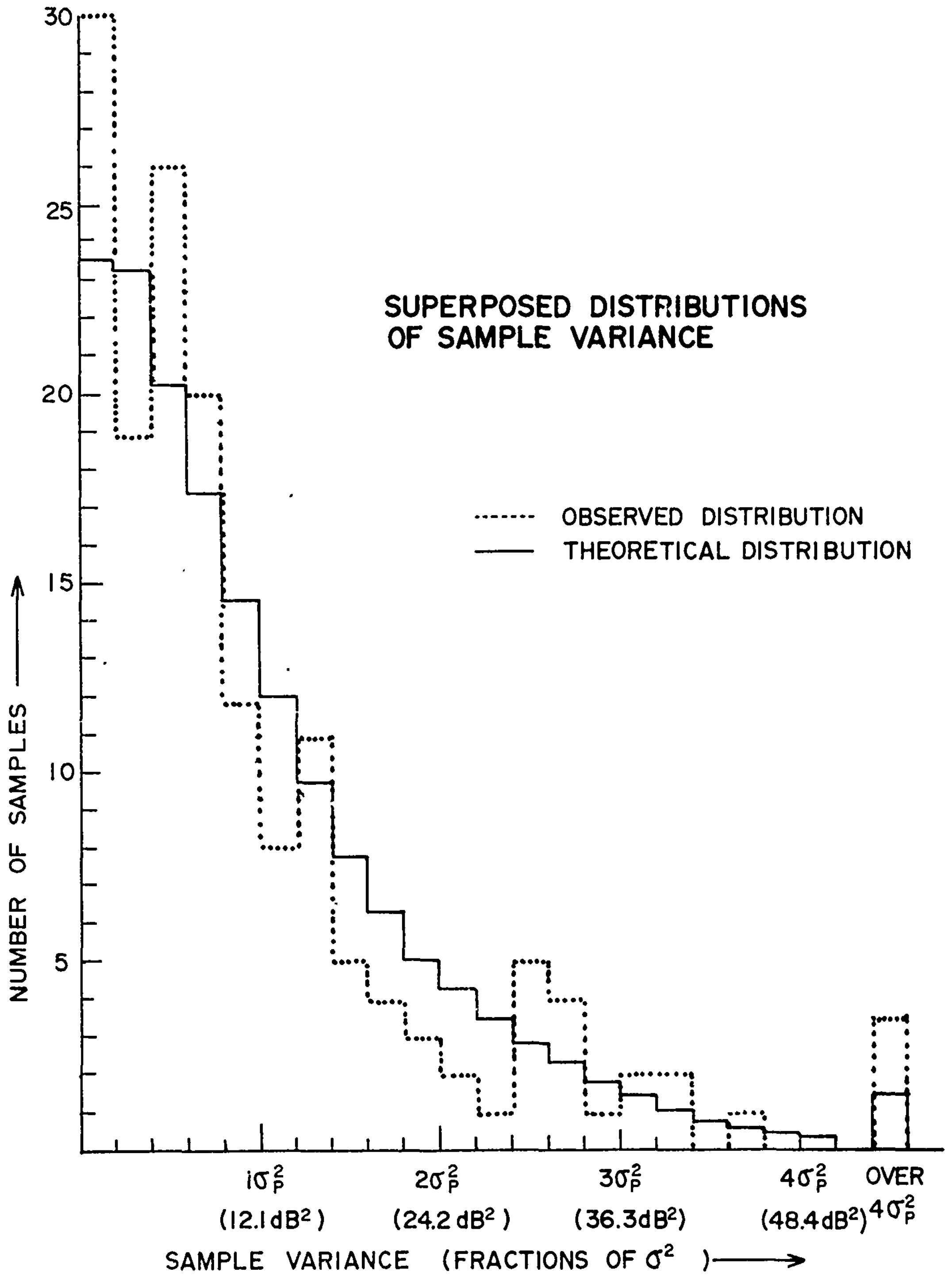
To test the hypothesis we first construct the theoretical distribution of sample variances to be expected if the true variance is indeed constant over all sample spaces. The next step is to compare the observed distribution of sample variance with the theoretical distribution. If substantial agreement exists, we conclude that the variance cannot be shown to vary from sample to sample.

The appropriate statistic to calculate the theoretical variance distribution is the "Chi square over df" (X^2/df) distribution. It is used to calculate the expected distributions of sample variance in 89 samples of size 3 and 71 samples of size 4. The two distributions are then summed. Results appear in Figure 9 where we see, for example, that 12 samples would be expected to have variances between 12.1 and 14.5 db^2. For convenient use of the X^2/df tables, the horizontal scale of the figure is broken into even fractions of Sp^2 rather than into whole db^2. We next determine the number of samples which actually do have variances in these intervals. The result is similar to Figure 8 except that the variance intervals (horizontal scale) have been changed to coincide with Figure 9. The observed distribution appears in Figure 10 along with the theoretical distribution superimposed. Figure 10 shows that the curves are roughly similar. To test for similarity, we use the chi - square one sample test which shows that the two distributions are not significantly different at the .2 level of significance. In summary, then, we accept the hypothesis that: The population variance is constant over the experiment area, and is equal to 12.1 db^2.



THEORETICAL DISTRIBUTION
OF SAMPLE VARIANCE
EXPECTED NUMBER OF SAMPLES
25
20
15
10
5
0
0
$1\sigma_P^2$
(12.1 dB^2)
$2\sigma_P^2$
(24.2 dB^2)
$3\sigma_P^2$
(36.3 dB^2)
$4\sigma_P^2$
(48.4 dB^2)
OVER
$4\sigma_P^2$
SAMPLE VARIANCE (FRACTIONS OF σ_P^2)

FIGURE 9



SUPERPOSED DISTRIBUTIONS
OF SAMPLE VARIANCE
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
NUMBER OF SAMPLES
30
25
20
15
10
5
$1\sigma_P^2$
(12.1 dB²)
$2\sigma_P^2$
(24.2 dB²)
$3\sigma_P^2$
(36.3 dB²)
$4\sigma_P^2$
(48.4 dB²)
OVER $4\sigma_P^2$
SAMPLE VARIANCE (FRACTIONS OF σ^2)

FIGURE 10

4. Distribution of Experimental Error. Since all of the statistical tests used in this report contain the implicit assumption that the population of measurements in each sample space is normally distributed, it is important to determine if this assumption is valid. The basic method of checking the data for normalcy of distribution is to calculate the theoretical distribution of single measurement deviations from the sample means which would be expected if the populations are indeed normal, and to compare this with the observed distribution of deviations from the sample means, for all samples.

To construct the theoretical distribution, we use the fact that for samples of size N from a normal population, the distribution of measurement deviations from the sample means (not the true means) is normal with a mean of zero and a variance of $\left(\frac{N-1}{N}\right) Sp^2$ (where Sp^2 is the population variance).

Thus, for a population variance (Sp^2) of 12.1 db^2 we obtain the following expected variances for samples of size N.

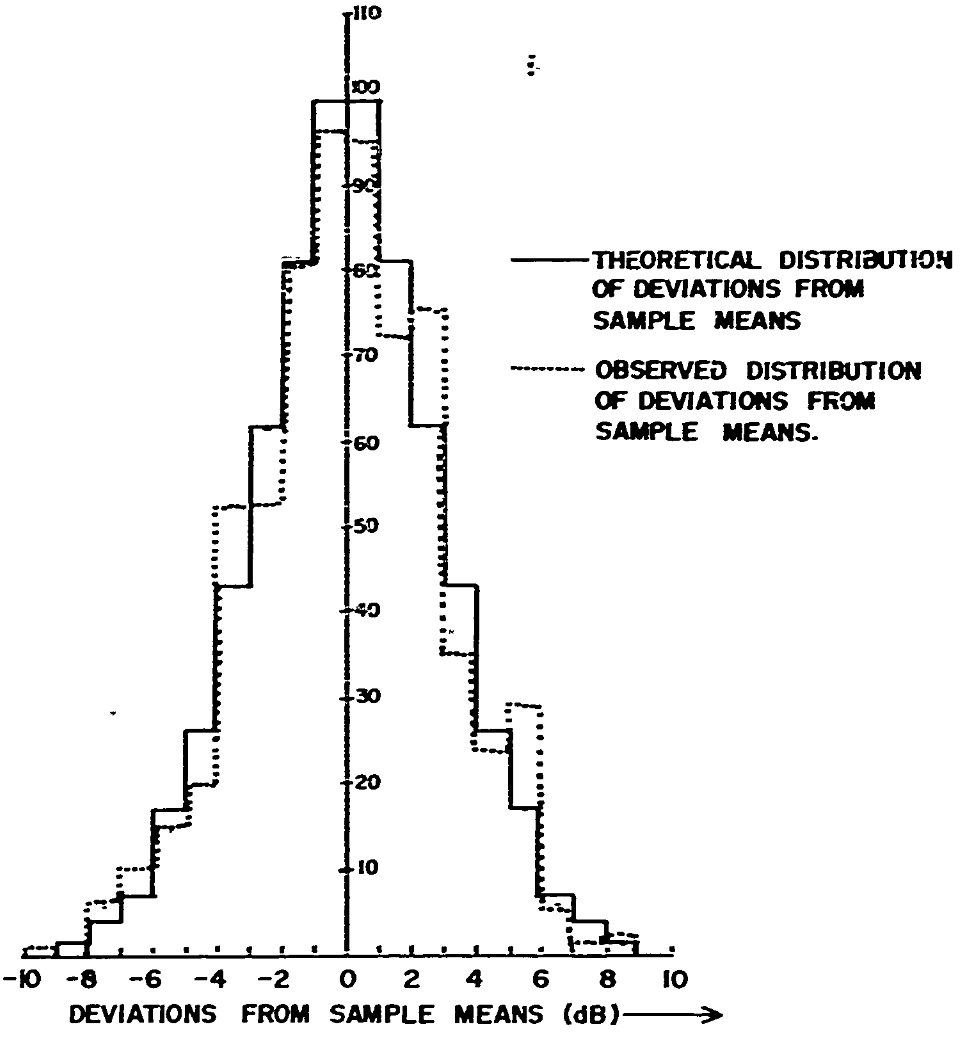
SAMPLE SIZE	s^2	s
N = 1	0	0
N = 2	6.05 db^2	2.46 db
N = 3	8.07 db^2	2.84 db
N = 4	9.08 db^2	3.01 db

Here, s^2 is the variance of the distribution of measurements about the sample means. And we have the following samples:

SAMPLE SIZE	NO. OF SAMPLES	TOTAL MEASUREMENTS
N = 1	(19)	19
N = 2	(52)	104
N = 3	(89)	267
N = 4	(71)	284
		674

The number of deviations which would be expected to fall into 1 db intervals is calculated for all samples, and the results are summed in Figure 11. For example, in a normal population we expect 99.7 deviations from sample means of between 0 and + 1 db and 26.2 deviations of + 4 to + 5 db.

Observed deviations are then also plotted in Figure 11 superimposed on the theoretical distribution. The figure reveals only minor differences. An appropriate test for equality of the two distributions is the Kolmogrov-Smirnov one sample test. To apply it we form the cumulative of each distribution and observe that the largest percentage discrepancy between the theoretical and observed cumulative distribution is 2.05%. For a sample size of 674, we accept the hypothesis of equality of distribution at the α = 20% level. The significance of this finding can be stated:



SUPERPOSED DISTRIBUTION OF DEVIATIONS FROM SAMPLE MEANS

FIGURE 11

The measurements obtained in this experiment do not demonstrate a departure from normal distribution. We have not "proved" that the data are normally distributed, only shown that they do not contradict the assumption.

In summary then, we accept the hypothesis that the experimental error is a normally distributed random variable with a constant standard deviation of 5.4 [illegible].

B. MODEL ERROR ANALYSIS.

1. General.

On first considering the experiment results and the model predictions in Section III-A we observed that the model appeared to predict too little loss especially at longer ranges. Also, the sample means were scattered about the model to some extent.

At this point, after analyzing experimental errors and model-data discrepancies, there remains considerable uncertainty concerning the model itself.

a. When differences occur between the model and the data, can the difference be attributed solely to sampling error in the data or is there evidence of significant model error?

b. How large a difference must occur before we can assert with some certainty that model error exists?

c. When there is a substantial bias difference between the model and the data, but the model seems to resemble the shape of the data curve (such as the buoy A results), can we measure the degree of correspondence in the shape of the two curves.

d. Where the model runs generally through the data, as in the case of buoys C and F, is the model an acceptable representation of the data?

Having investigated the experimental error of the measurements, we are now in a position to answer these questions and to investigate the model error.

2. Test for Presence of Model Error.

The basic method for determining if model error is present will be statistical hypothesis testing. In each sample space we have a number of measurements, the sample mean, a knowledge of sample variance, and a model prediction. We will formulate hypotheses concerning the model and the data, and through appropriate tests either accept or reject the hypotheses for each sample space. We then have, for each sample space, a decision as to whether or not the model contains significant error in that interval. A judgment that the model, as a whole, contains significant error is based on the relative number of samples which do and do not show model error. The first step is to test for agreement between the model and the hypothetical "true mean". To do this we form the hypothesis that: In each sample space the model value is equal to the true mean loss for that sample space. This does not mean, of

course, that the model value and the sample means must be identical. Rather we assume that, while we do not know what the true sample means are, we do know how the measurements are distributed about the true mean. So we hypothesize that the model value is the true mean in each sample space. If the distribution of measurements in a sample is far from the model, we will conclude that the model value is not the true mean in that smaple space, and reject the hypothesis that it is. This test is repeated in each sample space.

The appropriate test is the t - test for equality between a sample mean and a hypothetical mean.

The test will be applied twice to each sample, once with a level of significance (α) equal to 10% and once with α = 1%. The acceptance limits for the difference beteen model and sample mean from the "t" distribution are shown in Table 2. If model and sample means differ by more than these amounts, the model is rejected as being the true mean in that sample space.

SAMPLE SIZE	α = 10%	α = 1%
N = 1	5.75 db	8.90 db
N = 2	4.07 db	6.30 db
N = 3	3.31 db	5.11 db
N = 4	2.88 db	4.45 db

ACCEPTANCE LIMITS FOR MEANS TEST

TABLE 2

The results of each test are shown in Appendix A, arranged by sample; a tabulation of the results by buoy is shown here in Table 3.

BUOY	RANGE INTERVAL	SAMPLES	α = 10%		α = 1%	
			ACCEPT	REJECT	ACCEPT	REJECT
C	0-40 mi	39	16	23	28	11
F	0-40 mi	36	18	18	24	12
B	35-75 mi	40	22	18	33	7
G	35-75 mi	38	6	32	13	25
A	75-115 mi	38	3	35	15	23
H	75-115 mi	40	2	38	6	34
TOTAL		231	67	164	119	112

TABULATED RESULTS OF MEANS TESTS

TABLE 3

These results, of course, are not intended solely to indicate acceptance or rejection of the model in its entirety, but also to indicate areas of agreement and disagreement. The significant results of the tests appear in the columns headed "REJECT", which indicates the number of sample spaces in which the model was rejected as the true mean. For example, we see that of the 38 samples taken at buoy A, 35 resulted in rejection of the model at the 10% level.

It is important that the rejections be properly interpreted. A rejection at the 10% level means that, in that sample space, the model was sufficiently different from the sample mean that the difference could not be attributed solely to sampling error. The 10% significance level means only that the rejection has a 10% or less chance of being wrong, and the model actually being equal to the true mean. Rejection of the model at the 1% level is consequently a much more certain rejection, admitting only a 1% chance of being wrong.

As to the overall correspondence of the model and the data, there are numerous samples where model error is shown to exist with a high degree of certainty. The model was rejected in over 70% of the sample spaces at the 10% level and in over 48% at the 1% level.

It must be emphasized strongly, however, that these results reflect primarily on the certainty with which the rejections are made and give little information on the size or distribution of differences between the model and the true means.

Another factor requires consideration in interpreting Table 3. As it happens, the model is rejected so frequently that there is little question of certainty. But had the results been less overwhelming we would have to consider how many rejections should occur if the model were indeed perfect. When we make a number of tests at the 10% level, we accept the fact that we will be wrong about 10% of the time. Consequently, if we make 231 tests as we did here, we expect to get about 23 rejections if the model is perfect. If our total number of rejections had been much closer to 23 instead of 164 we would have to consider the possibility of erroneously rejecting a "good" model.

Another perspective of these results is presented in Figures 12 through 17. Each contains a loss versus range plot for results obtained at one buoy. Sample means and the model are plotted on each. About 40 sample spaces are represented on each curve. There are a number of interesting though qualitative features in these figures.

BUOY C: The model here is not badly biased and appears to run thru the data fairly well. The "t" tests, however, show only moderate regions of statistically acceptable agreement, for the data excursions are sufficient to indicate significant model error.

BUOY F: Agreement is fair. The model did not reflect the regions of high loss between 15 and 20 miles and between 27 and 40 miles.

BUOY B: Model-data agreement was closest in results from this buoy. A remarkable shape correspondence exists between 57 and 70 miles.

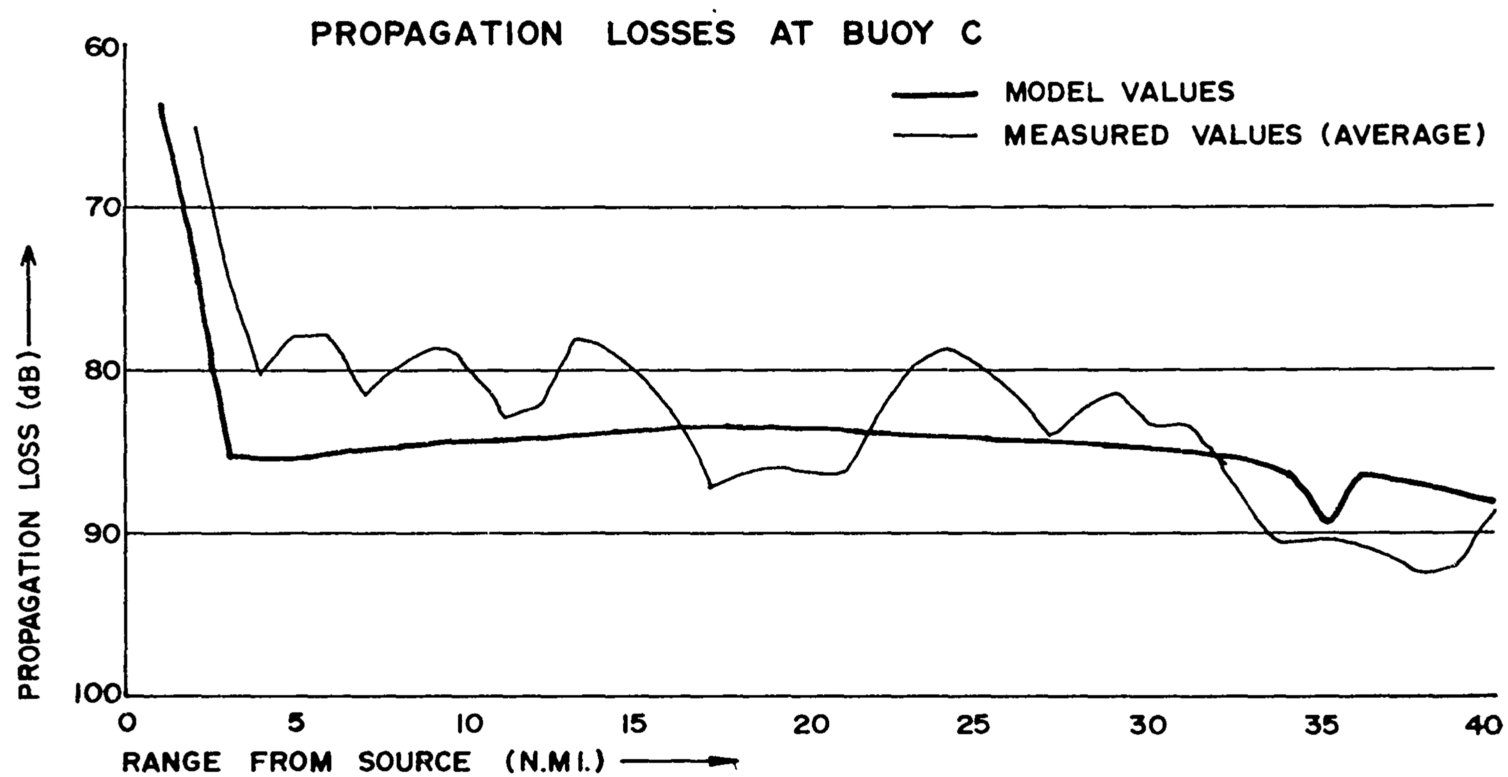

PROPAGATION LOSSES AT BUOY C
MODEL VALUES
MEASURED VALUES (AVERAGE)
PROPAGATION LOSS (dB)
60
70
80
90
100
0
5
10
15
20
25
30
35
40
RANGE FROM SOURCE (N.M.I.)

FIGURE 12

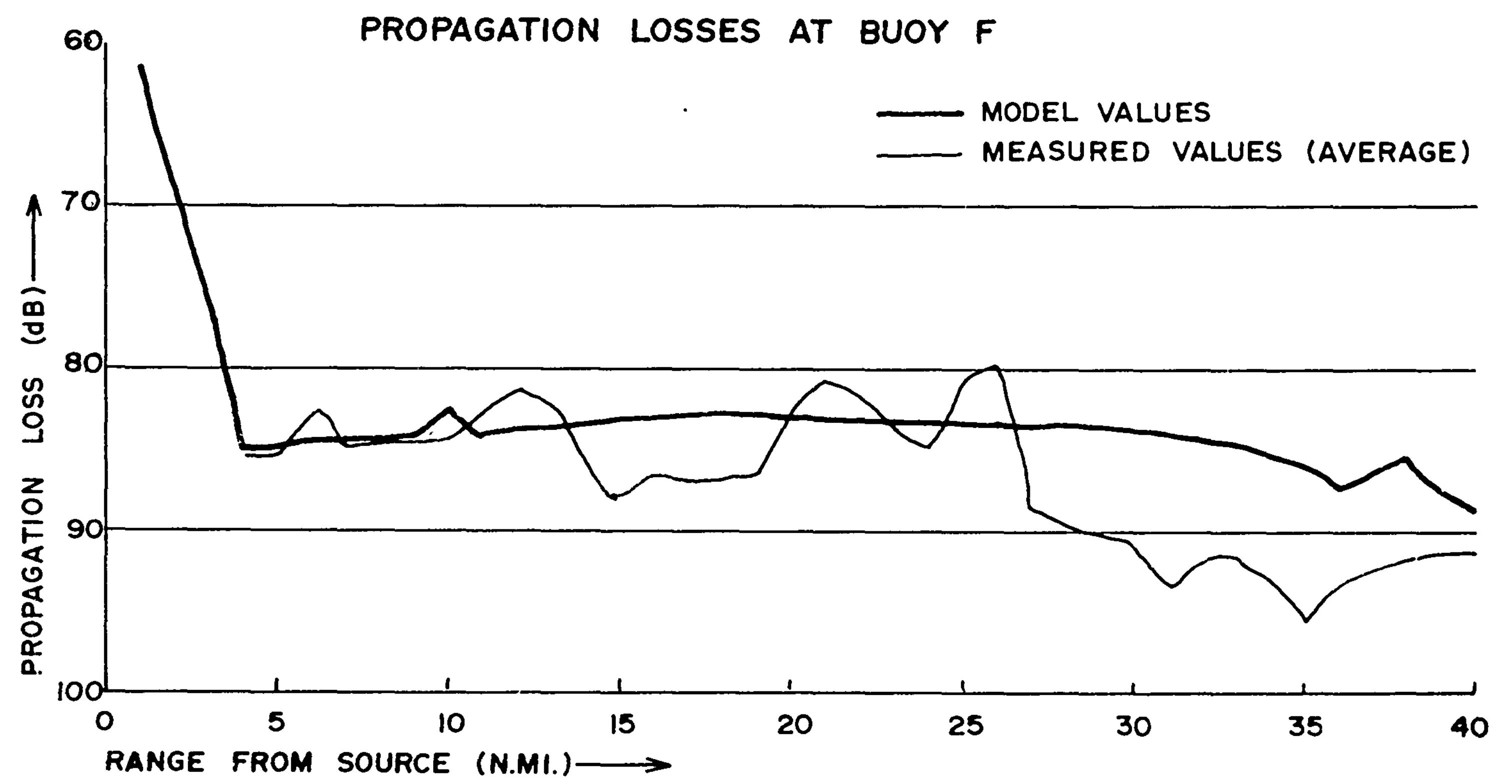

PROPAGATION LOSSES AT BUOY F
MODEL VALUES
MEASURED VALUES (AVERAGE)
60
70
80
90
100
PROPAGATION LOSS (dB)
0
5
10
15
20
25
30
35
40
RANGE FROM SOURCE (N.MI.)

FIGURE 13

PROPAGATION LOSSES AT BUOY B

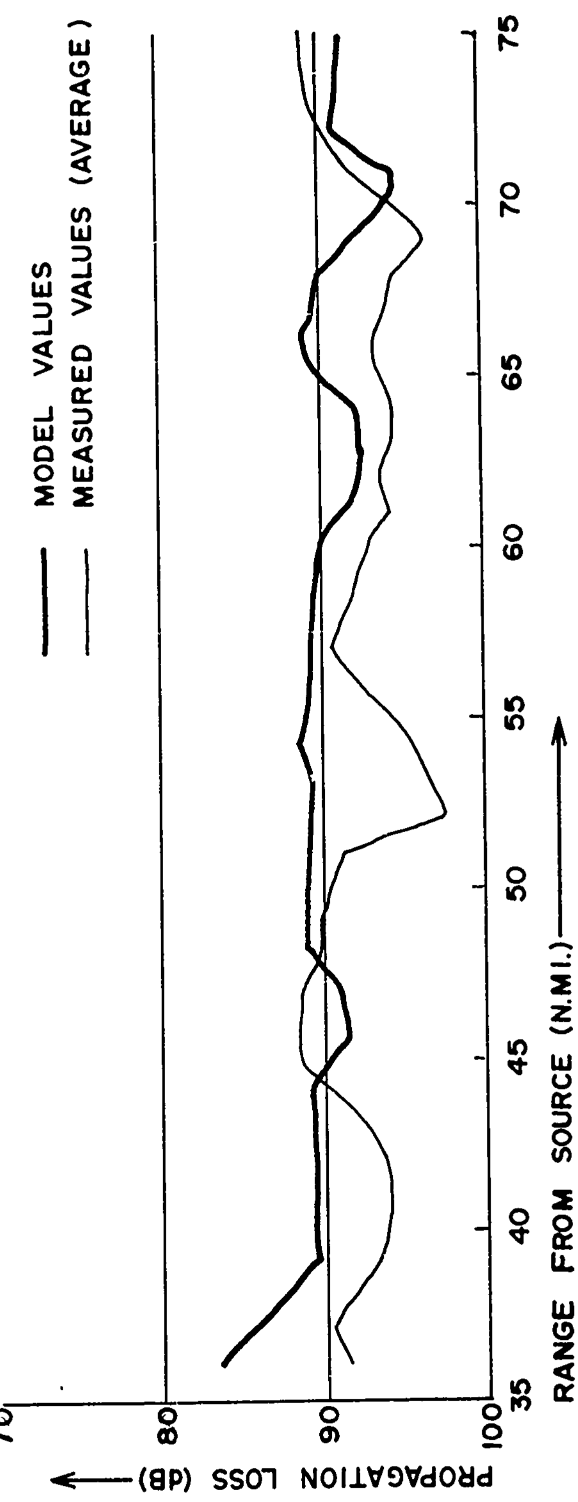


FIGURE 14

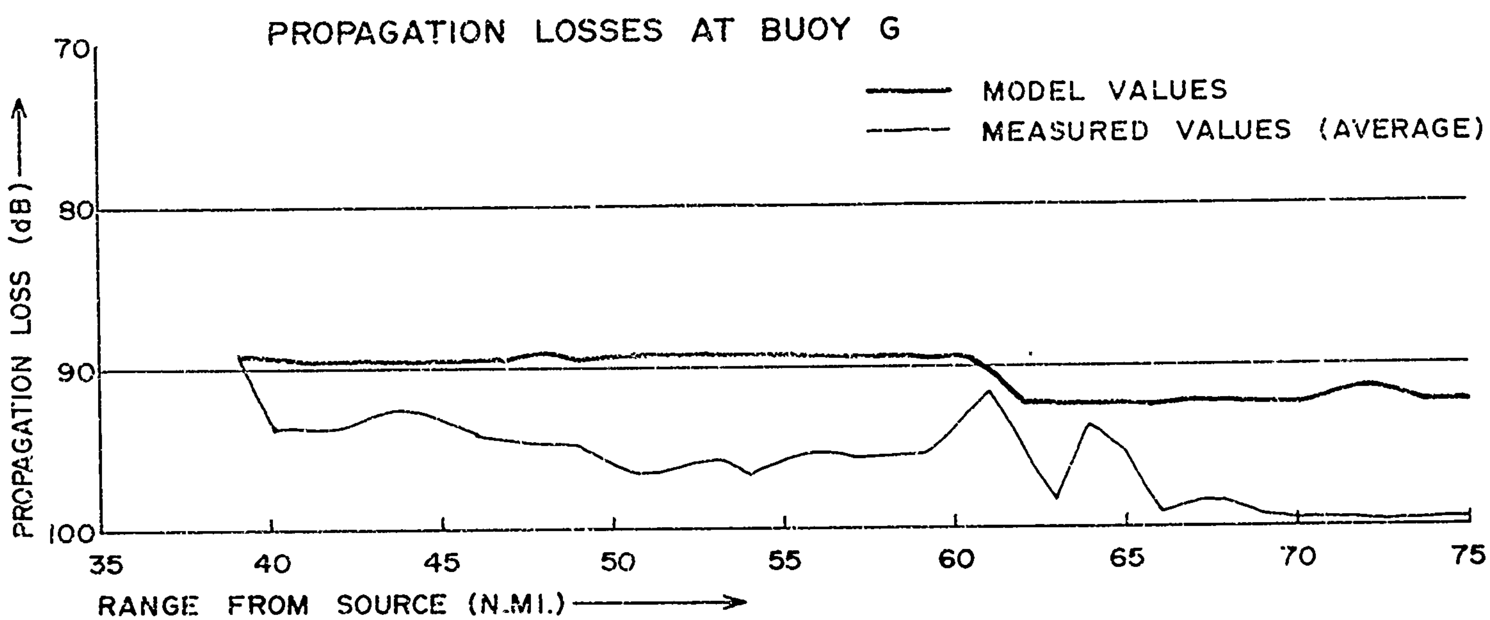
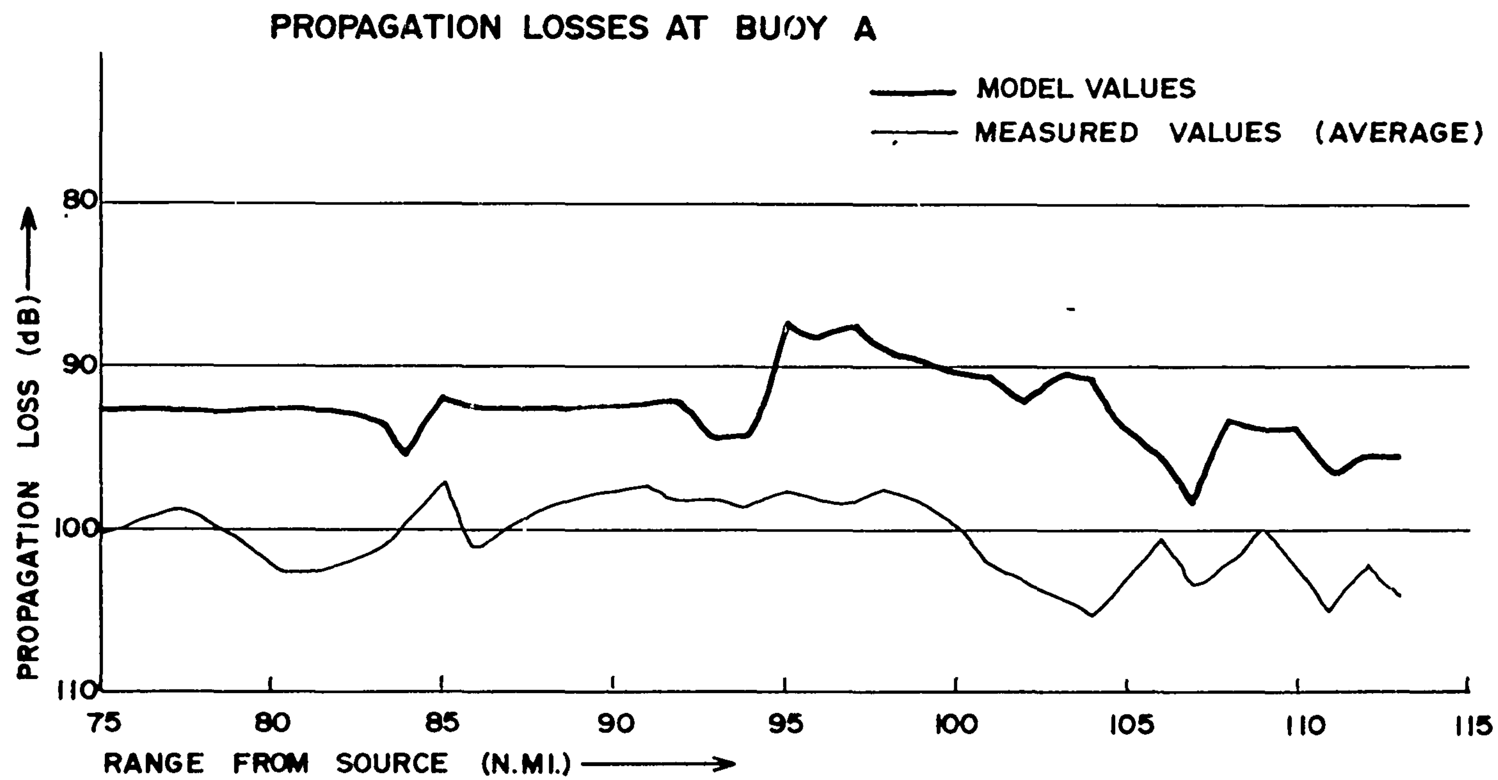

PROPAGATION LOSSES AT BUOY G
MODEL VALUES
MEASURED VALUES (AVERAGE)
PROPAGATION LOSS (dB)
70
80
90
100
35
40
45
50
55
60
65
70
75
RANGE FROM SOURCE (N.MI.)

FIGURE 15



PROPAGATION LOSSES AT BUOY A
MODEL VALUES
MEASURED VALUES (AVERAGE)
PROPAGATION LOSS (dB)
80
90
100
110
75
80
85
90
95
100
105
110
115
RANGE FROM SOURCE (N.MI.)

FIGURE 16

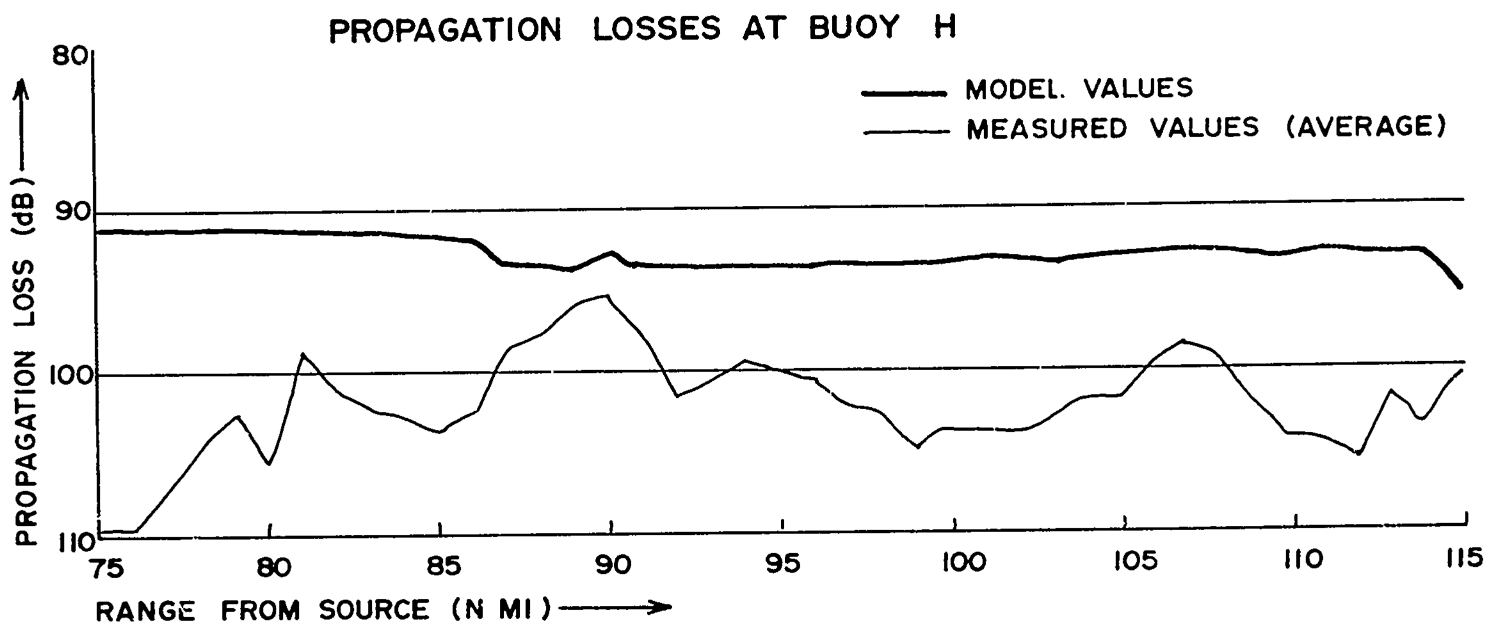

PROPAGATION LOSSES AT BUOY H
MODEL. VALUES
MEASURED VALUES (AVERAGE)
80
90
100
110
PROPAGATION LOSS (dB)
75
80
85
90
95
100
105
110
115
RANGE FROM SOURCE (N MI)

FIGURE 17

BUOY G: The model consistently showed about 5 db too little loss. Both curves are relatively flat with fair shape correspondence.

BUOYS A, H: The model was consistently off 8 to 10 db here at long range. There is some shape correlation at buoy A, but little at buoy H. Both buoys were the same distance from the target (75-115 mi) but the model predicts somewhat different results for losses up slope to buoy A and down slope to buoy H.

5. Components of Model Error.

The previous sections have established the satistical certainty that model error exists and have investigated the character of the experimental error. But we have not yet tried to describe the model error or that error between the model and the true means.

Since, in any particular sample space, the true mean loss is unknown, we will not be able to determine the model error present in any single sample space. But it is possible to determine some characteristics of the model error by considering error distributions.

Before proceeding, it will be helpful to look more closely at how "model error" may be described. Measurements reveal that the model persistently predicted too little loss over considerable range intervals. This leads to describing the model error in two components: The "bias error" and what will be called "tracking error". These two components are not associated with different physical processes; the separation is only for mathematical convenience. The components will have separate significance to model designers only if they can be associated with distinct parts of the propagation loss model. The separation is made for convenience of description and may be interpreted variously by various parties.

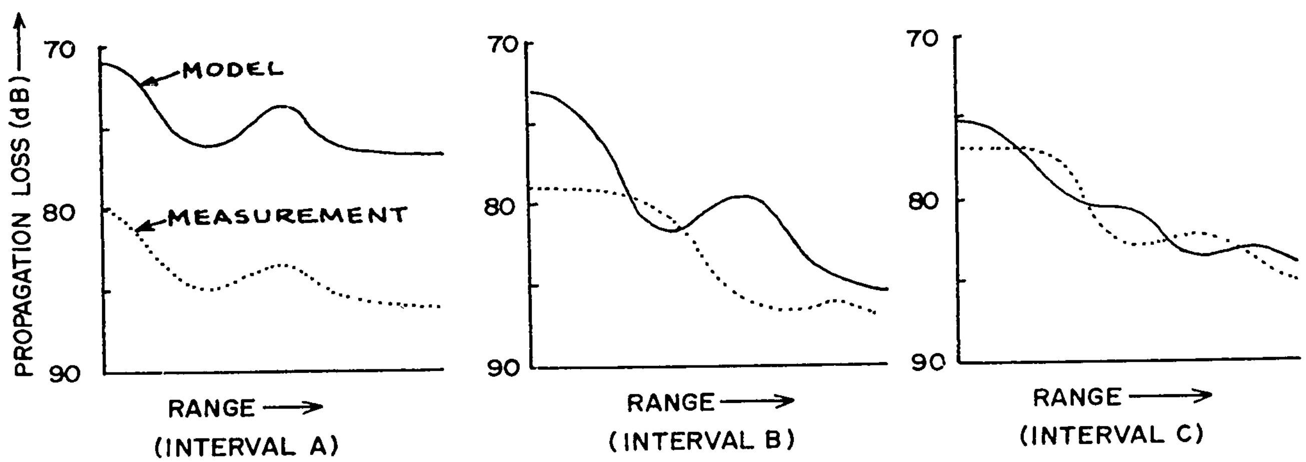
The "bias error" is defined as the mean model error over a number of sample spaces. It only describes model error over some interval of sample spaces, and its numerical value will depend on the interval over which we calculate it. We have chosen, rather arbitrarily, to calculate the mean bias over sets of sample spaces associated with each buoy. Model bias in the interval 0 to 40 miles is given by bias in the buoy C and buoy F samples; model bias from 35 to 75 miles is indicated by buoy B and G samples; model bias from 75 to 115 miles is indicated by buoy A and H samples.

The "tracking error" represents a fluctuating error superimposed on the bias. Combined, the two produce model error.

The relation is represented graphically in Figure 18 which shows hypothetical model and loss curves. In interval A there is about 10 db bias error between the model and the actual loss curve. But tracking error is very small for when the bias is removed the two curves match well.

In interval B there is about 4 db bias and also significant tracking error. When the bias is removed the curves differ significantly.

In interval C there is little or no bias but significant tracking error.



HYPOTHETICAL PROPAGATION LOSS CURVES

FIGURE 18

In summary, "tracking error" provides some measure of how well the shape of the model loss curve agrees with the shape of the true loss curve and "bias error" indicates how much the two curves are separated in an interval.

We now want to estimate the values of the two components. Bias has already been determined in Section III-B but there it was calculated as the mean deviation of the model from the sample means, not the true means as we require. If we assume, however, that the sample means are unbiased estimates of the true means, the bias figures represent the same value. The assumption of unbiased samples means we assume the measuring and processing equipment introduced only random errors with zero mean and did not consistently raise or lower measurements and that the samples obtained were randomly selected from all measurements which might have been made moments earlier or later. In the absence of contrary evidence, we adopt these assumptions. Thus, the bias indicated in samples at each buoy is:

BUOY C: + 1.9 db BUOY F: - 2.5 db	0 to 40 miles
BUOY B: - 2.8 db BUOY G: - 5.7 db	35 to 75 miles
BUOY A: - 8.2 db BUOY H: - 9.3 db	75 to 115 miles

We now want to estimate the model tracking error. To begin with, we have the distribution of measurement deviations from the model in Figure 19 which shows how many measurements differed from the model by a given amount. Each of the deviations is a random combination of experimental and model errors. One of the 10 db deviations, for example, might have occurred because a measurement was 95 db, the true mean 90 db, and the model 85 db. The 10 db combined deviation is made up of 5 db sampling error and 5 db model error.

The first step in separating the combined errors is to remove the model bias error. Graphically, this involves repositioning the model trace on the loss curve so that the model runs generally through the data. Mathematically, we de-bias the model by applying the appropriate bias factor to the model value in each sample space. The effect is to remove the bias error and produce Figure 20 which shows the deviation of measurements about the de-biased model. In this figure, each of the deviations is composed only of sampling error and tracking error. And if we can "remove" the sampling error from this distribution, the tracking error alone will remain.

The sampling error was found, in Section III-C, to be a normally distributed random variable with zero mean and a standard deviation of 3.48 db The theoretical distribution of sampling errors in 674 measurements may be constructed as in Figure 21 which shows the <u>expected</u> distribution of sampling errors. There is, of course, no way of determining the exact sampling errors encountered in the experiment. For purposes of estimating the tracking error, we will assume that the theoretical distribution of sampling error is an adequate representation of actual sampling errors.

To obtain an estimate of the tracking error, we reason as follows: Figure 20 is the distribution of sampling error plus tracking error and Figure 21

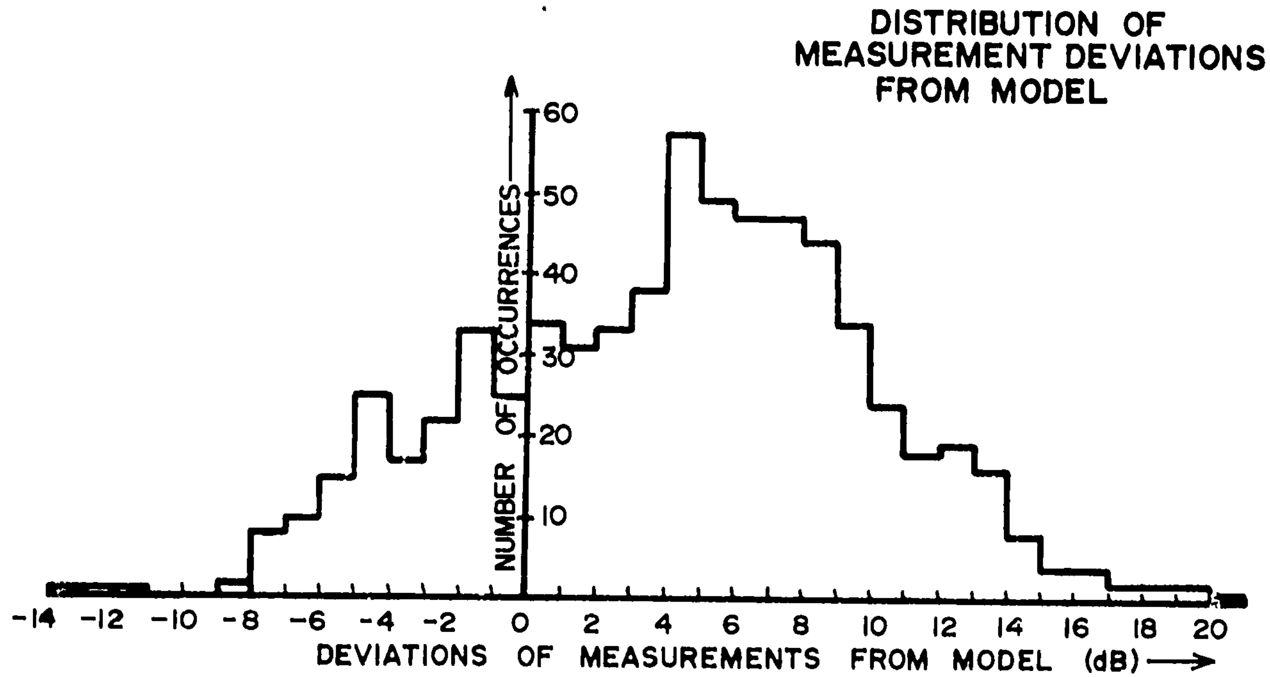

DISTRIBUTION OF
MEASUREMENT DEVIATIONS
FROM MODEL
NUMBER OF OCCURRENCES
60
50
40
30
20
10
-14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18 20
DEVIATIONS OF MEASUREMENTS FROM MODEL (dB)

FIGURE 19

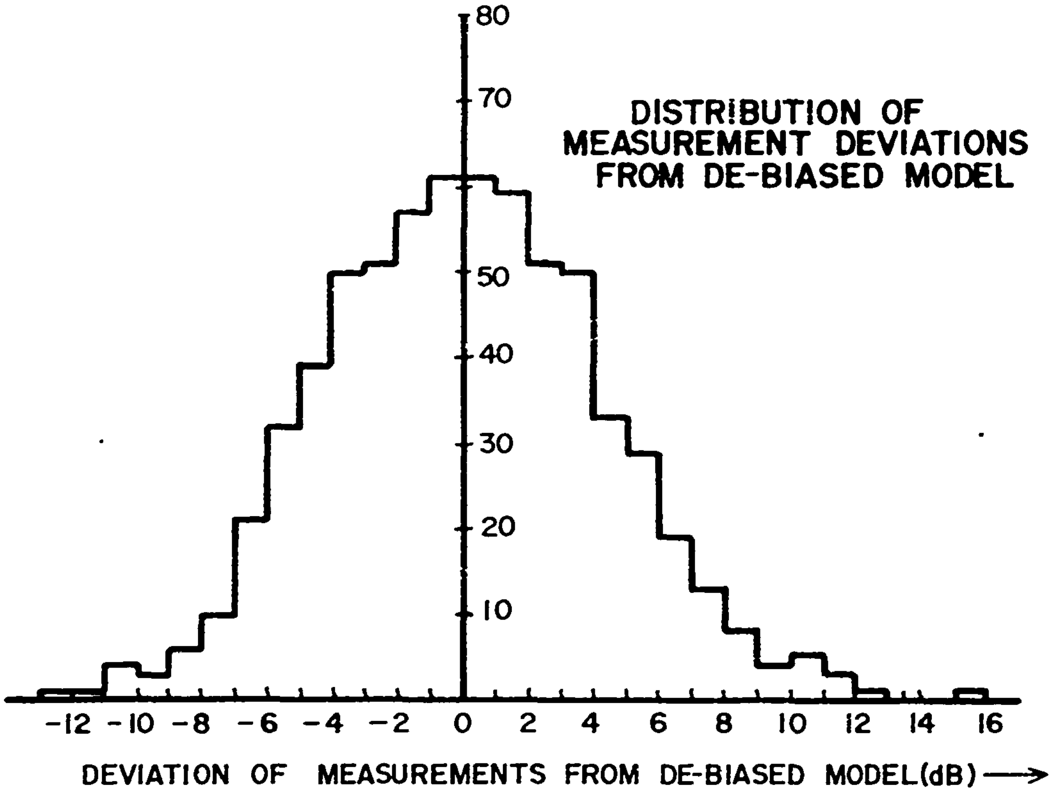
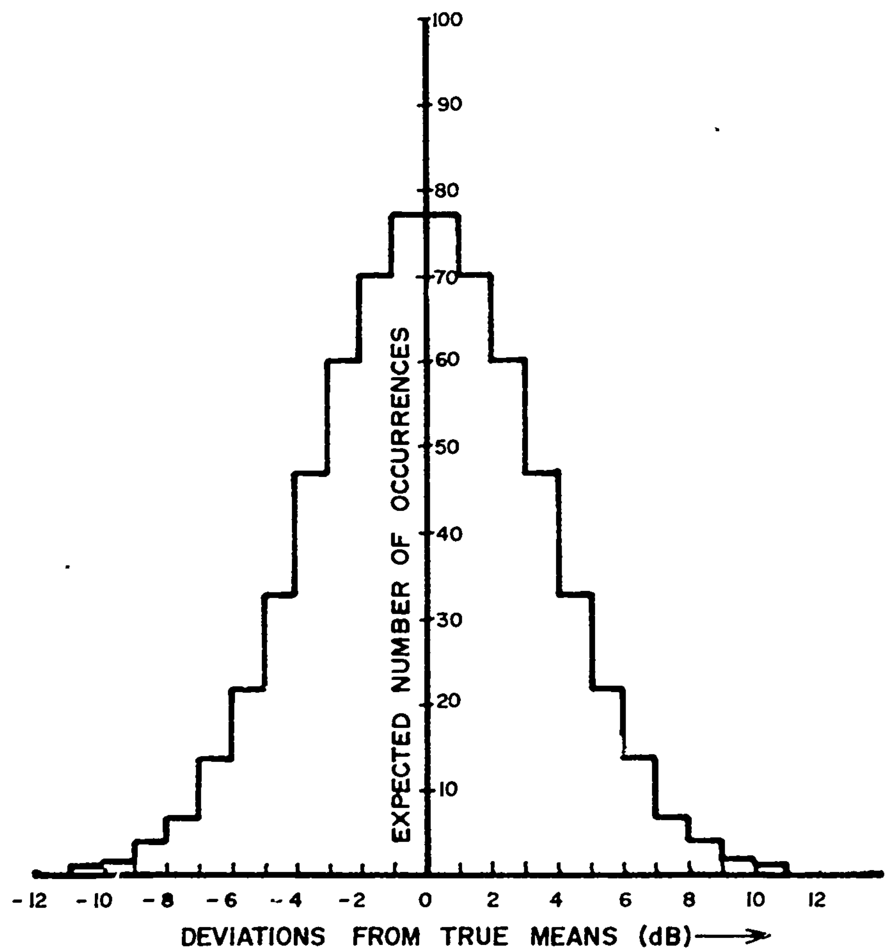

80
70
50
40
30
20
10
DISTRIBUTION OF
MEASUREMENT DEVIATIONS
FROM DE-BIASED MODEL
-12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16
DEVIATION OF MEASUREMENTS FROM DE-BIASED MODEL(dB)

FIGURE 20



THEORETICAL DISTRIBUTION OF MEASUREMENTS ABOUT TRUE SAMPLE MEANS

FIGURE 21

is the distribution of sampling error alone; the difference is due to the presence of tracking error. The two are shown superimposed in Figure 22.

There are a number of distributions of tracking error which could be added to the sampling error to produce approximately the results observed in Figure 20. But as the resultant is approximately a normal distribution, we hypothesize that the model tracking error is also normally distributed and independent of sampling error.

Figure 20, then, is hypothesized to be the distribution of the sum of two normally distributed random variables. For the sums of independent normal random variables we have the relation:

$$\sigma_s^2 + \sigma_t^2 = \sigma_\Sigma^2$$

or

$$\sigma_t = \sqrt{\sigma_\Sigma^2 - \sigma_s^2}$$

Where:

σ_Σ^2 = Variance of summed sampling and tracking errors

σ_s^2 = Variance of sampling errors

σ_t^2 = Variance of tracking errors

We also know σ_s = 3.48 db and σ_Σ was calculated to be 4.25 db. (Standard deviation of Figure 20). Solving the latter equation:

We have: σ_t = 2.45 db.

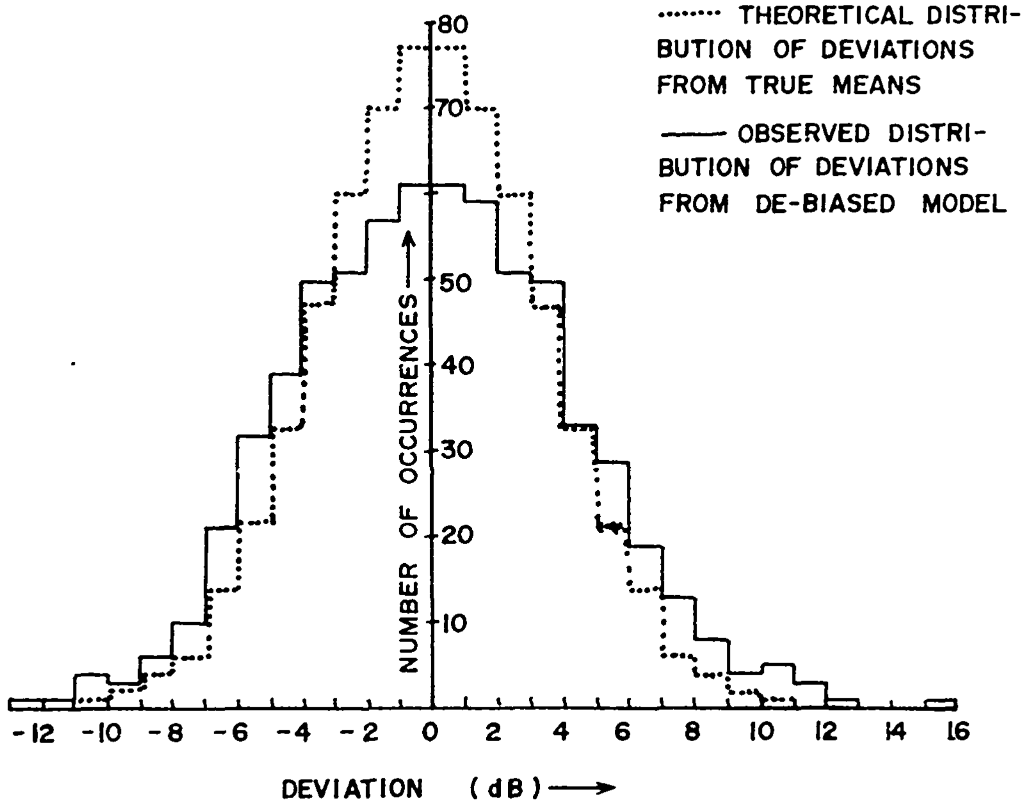
So if the tracking error is normally distributed, it has a standard deviation of 2.45 db.

In summary, the model error, for descriptive purposes, consists of two components: bias error and tracking error. The bias error increases with range from 0 at close range to 8 or 9 db at 100 miles. The tracking error is superimposed on the bias. It is of unknown distribution, but if assumed to be normally distributed it has a standard deviation of 2.45 db and a mean of zero.

E. INFLUENCE OF BOTTOM TOPOGRAPHY.

1. General.

The present version of the propagation loss model has provision for making use of bottom topography data. For this project, bottom depth at half-mile intervals along the line of buoys was introduced to the program. The program interpolated linearly between these points to construct a continuous bottom profile. Bottom depth and slope were taken from the linear segments to calculate depth of reflection points and angles of reflection from the bottom.



SUPERPOSED DISTRIBUTIONS OF DEVIATIONS

FIGURE 22

In this section of the report, we consider the influence of bottom topography on the model predictions, on the observed data, and on the correspondence between the two.

Bottom topography is one of the "controlled" variables of the experiment and is controllable in the model. This does not mean that some conceptual bottom profile can be arbitrarily chosen for the experiment, but only that each sample consists of measurements which were nominally made over the same bottom segment and that the shape of this bottom segment can be specified to the model.

Due to the geometry of the experiment, we have two sets of paired samples. One set is the up slope measurements and one set down slope. The two members of each pair were made under conditions which were identical except for the bottom profile; other controlled variables such as range and velocity profile were nominally identical for both members of the pair. Thus, when differences occur between members of a pair, the difference must be due only to bottom difference and sampling error and not to variation of other controlled variables.

We note also that loss predictions from the model are similarly paired. We have one curve for losses up slope, toward buoys A, B and C and one curve for down slope losses to buoys F. G and H and for any range we have two model predictions.

2. Bottom Topography and the Model.

We look first at what the model predicts the influence of differing topography will be. In Figure 23 the predictions for up slope losses are superimposed on down slope loss predictions. We see that the two curves correspond quite closely out to a range of about 95 miles. To 20 miles the difference is expected to be generally less than one db and from there to 95 miles, the expected difference is generally one to two db. We infer from this figure that the model predicts that bottom topography influence will be minimal at near and medium ranges and will increase to some extent at ranges over 95 miles.

3. Bottom Topography and Loss Data.

The observed loss measurements are shown in Figure 24. One trace is the sample means in the up slope direction. Superimposed is a trace of the sample means down slope. Figure 24 contains some interesting, though qualitative, features.

First, there is much less agreement between those two curves than between the two model traces. This is to be expected, however, since the sampling error is included in the observed loss curves.

We see that the curves agree better at long range than at short range which is just the opposite of what the model predicts.

Since Figure 24 is a plot of sample means, it includes the sampling error and some part of the observed differences in the two curves is due to this factor. The question thus arises whether bottom topography caused any difference at all in the two sets of measurements or if the differences may be due entirely to sampling error. To answer this question

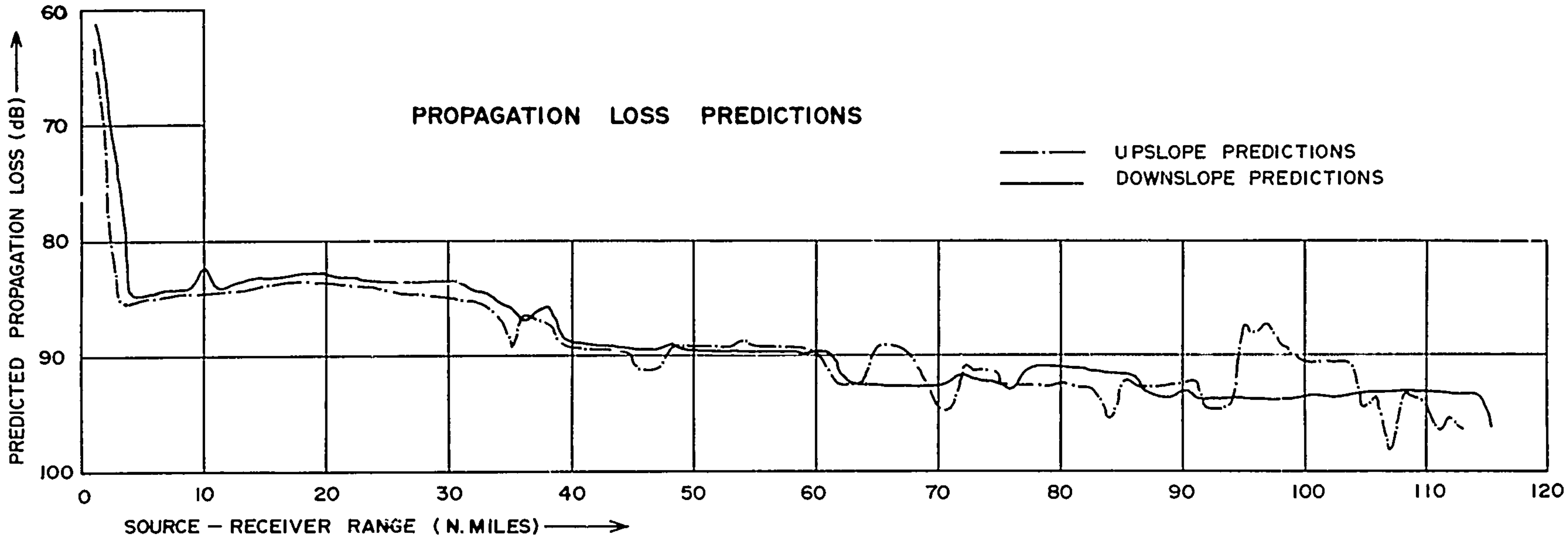

PROPAGATION LOSS PREDICTIONS
UPSLOPE PREDICTIONS
DOWNSLOPE PREDICTIONS
PREDICTED PROPAGATION LOSS (dB)
60
70
80
90
100
0
10
20
30
40
50
60
70
80
90
100
110
120
SOURCE – RECEIVER RANGE (N. MILES)

FIGURE 23

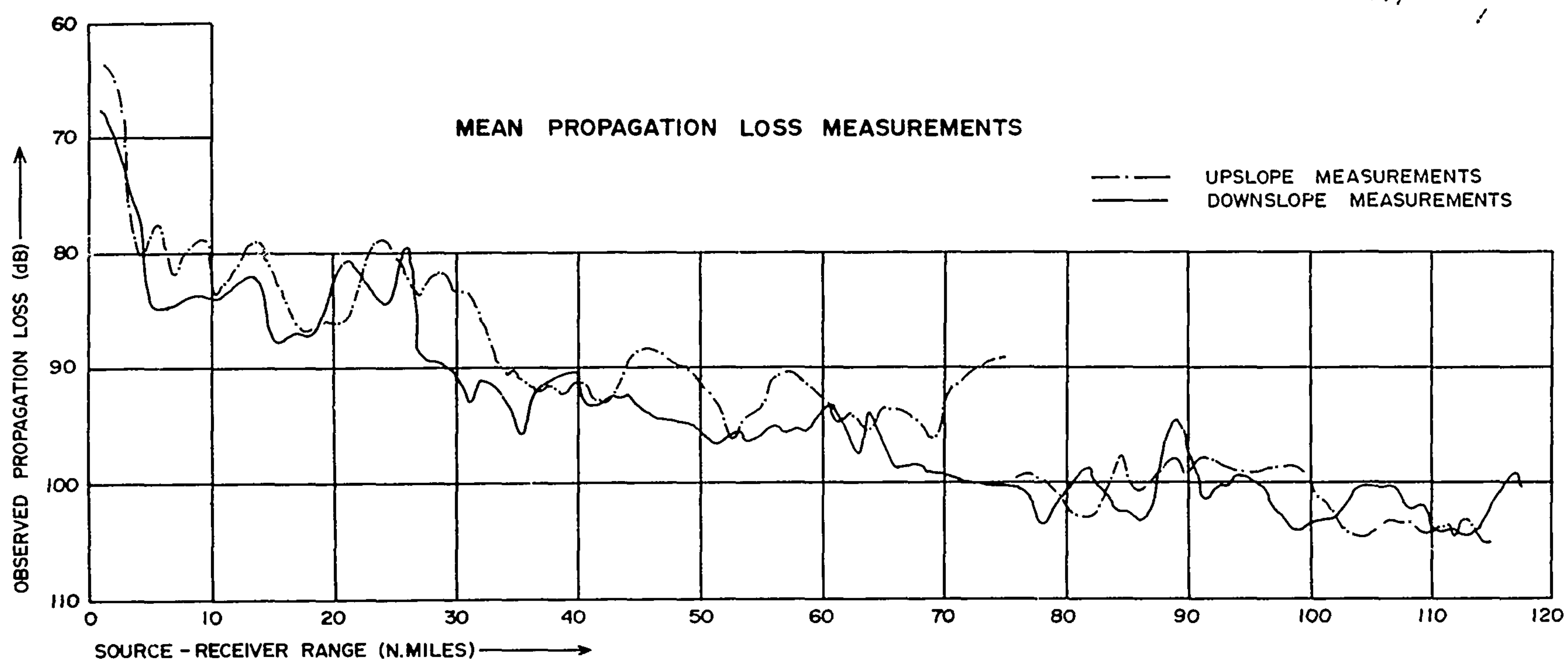

MEAN PROPAGATION LOSS MEASUREMENTS
UPSLOPE MEASUREMENTS
DOWNSLOPE MEASUREMENTS
OBSERVED PROPAGATION LOSS (dB)
60
70
80
90
100
110
0
10
20
30
40
50
60
70
80
90
100
110
120
SOURCE - RECEIVER RANGE (N.MILES)

FIGURE 24

we use a hypothesis testing procedure similar to that employed to compare the model to the sample means.

We formulate the hypothesis: The two members of each sample pair were drawn from populations which have equal means. If accepted, this hypothesis would indicate that there is no significant difference between up slope and down slope loss measurements and that differences are due only to sampling error. We use again the "t" test for equality of means.

$$|\bar{x}_1 - \bar{x}_2| \leq Sp\ (1/N_1 + 1/N_2)\ t_\alpha \qquad \text{with } \alpha = .1$$

Where:

$\bar{x}_1$ = Sample mean for first sample of a pair

$\bar{x}_2$ = Sample mean for second sample of a pair

Sp = Pooled variance of 231 samples

N_1 = Sample size of first sample

N_2 = Sample size of second sample

t_α = "t" for α = .1 and 391 degrees of freedom

The test is applied to each sample pair and the hypothesis is accepted if the left hand side of the equation is less than the right. Acceptance limits for various combinations of N_1 and N_2 are shown in Table 4.

N_1 \ N_2	1	2	3	4
1	8.0 db			
2	6.95 db	5.69 db		
3	6.55 db	5.18 db	4.54 db	
4	6.37 db	4.92 db	4.05 db	4.02 db

ACCEPTANCE LIMITS FOR MEANS TESTS

TABLE 4

The outcome of these tests for each sample pair is summarized here:

Total pairs tested	110
Pairs with equal means	73
Pairs with unequal means	37

Of the 110 sample pairs tested, 37 were found to have unequal means. Recalling that we used a significance level of α = 10% we expect to get about 11 rejections even if all pairs really do have equal means. However, the occurrence of 37 rejections is a strong indication that all pairs do not have equal means and we reject the hypothesis that they do.

We conclude that some of the difference between up slope and down slope measurements is due to differing bottom topography and is not due entirely to sampling error.

A question naturally arises at this point concerning how much of the difference observed in the curves is due to differing bottom profiles. And it would be very useful to separate the influence of bottom difference and sampling error. Unfortunately, the sample sizes are simply too small to separate the two effects. We can say with certainty that the two effects co-exist in the data, but that they cannot be separated and measured.

4. Model Success with Bottom Profile.

We come next to the question of how well the model made use of the bottom topography data. Can it be shown that use of bottom profile improved the model predictions?

Since we are unable to say how much of the difference between two measured loss curves is due to difference in bottom profile, it appears that we cannot determine how well the model used the profile data. For example, the model might predict a given difference between members of one pair. But when we look at the observed data we do not know how much of the observed difference is due to sampling error. And in 73 out of 110 pairs we cannot be certain that a significant difference really exists between members of the pair.

We are able to obtain one interesting result, however, by concentrating on the 37 instances where the measured losses were significantly different for up slope and down slope measurements. The data suggest that, at least at these points, the bottom caused a significant difference in losses. We see also that for these 37 pairs the model also makes different prediction for each member of the pair. A comparison of each of the measured loss pairs with the corresponding model values shows that in 24 of the 37 pairs the model was incorrect in predicting which member of the pair would show greater loss. It was correct in 13 cases.

Similar comparison for all 110 sample pairs show the model was correct 53 times and incorrect 57 times.

This is an interesting but inconclusive finding. In essence it implies that, had the model erroneously mixed the bottom profiles, better predictions would have resulted in this instance. We do not suggest that taking bottom profile into account is fruitless. We simply cannot show that the model prediction process was improved by using data on the minor differences between two similar bottom segments.

IV. INTERPRETATION AND IMPLICATIONS

To this point, the object of this analysis has been to investigate the relationship between the propagation loss model and a set of measurements. Quantitative descriptions of model and experimental errors have been presented. But these numerical results have varying significance and implications for various purposes. We will make some general observations and expect that interested parties will arrive at other interpretations appropriate to their own purposes.

The analysis describes two arbitrary components of model error. We do not suggest, however, that simply adjusting the model mechanically to reduce or eliminate these particular errors is desirable. The model has been designed to simulate a physical process and further refinement should proceed on the basis of research into the fundamental physical processes involved and perhaps into the stochastic behavior of some of the variables. The measurement of error is simply a criterion by which to measure progress; the character of the error may give little or no indication of the source of error.

The separation of experimental error from model error, however, indicates potential benefits which might be obtained by further model development. Figure 25 illustrates the distributions of errors which may be expected under three conditions. First, curve A indicates the distribution of combined sampling and model error for single observations. If, for example, the present model predicts a certain loss between two points, a 50% chance exists that a single measurement between those points will disagree with the prediction by 5.5 db or more.

Curve B represents the expected errors between single observations and a hypothetical model from which the bias has been removed. This curve has physical significance only if the model bias error results from some identifiable cause and can be removed without influencing the remainder of the variables. It is shown here to illustrate the strong influence of bias error in relation to tracking error.

Curve C represents the distribution of errors that would be observed between single measurements and a "perfect" model, i.e. one that exactly represents the true mean loss function.

If we assume that the model is used primarily to predict the loss in a single observation or in a number of observations in adjacent spaces, then we are interested in the errors shown in curve A, not merely in the model error alone. And we are interested in the reduction of total resultant error (model error plus sampling error) which occurs due to reducing the model error component. Thus, we see that improvement from the present model to a "perfect" model will reduce the total prediction error by about one-half. (Errors in curve A are roughly twice those in curve C.) We also see that developments which eliminate the model bias produce a model which is nearly perfect (curve B), and that failure of the model to follow minor fluctuations of the true loss contributes a very minor error increase. These observations, based on partial and presumptive evidence, are advanced as conjectures and not conclusions. So it is appropriate to review here some of the limitations of this analysis and their possible influence on the results. These are:

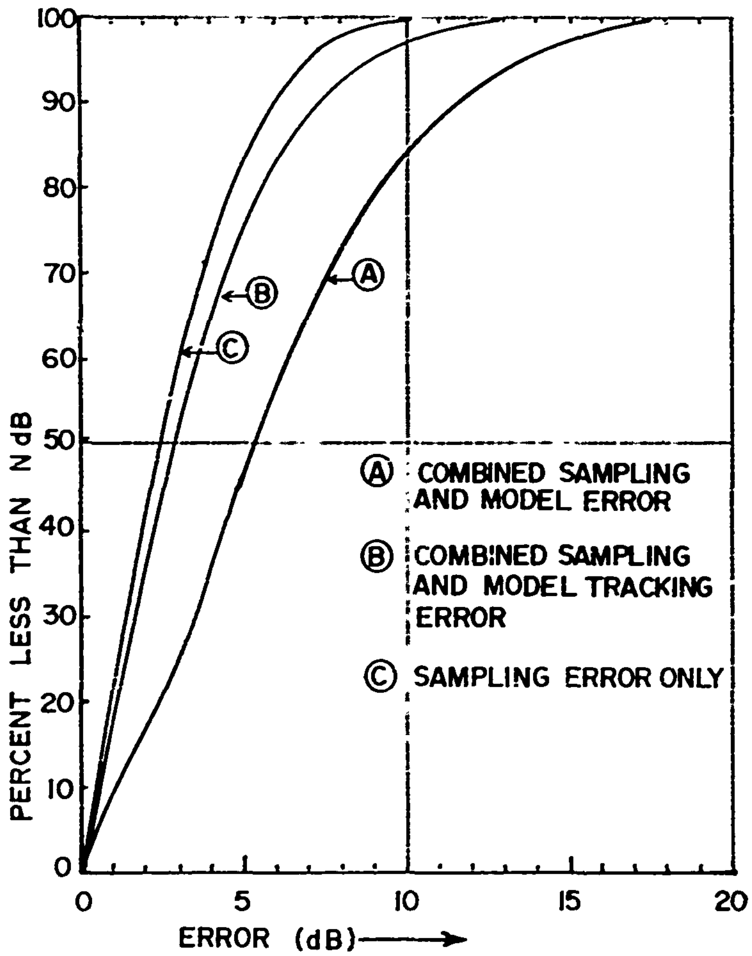
1. The greatest and most obvious limitation is the restricted area and time span of the experiment. These results are not necessarily representative of other areas.

2. A number of elements are grouped together into the single category of "experimental" or "sampling" error. We assume that the primary component is variation of the phenomenon being measured, i.e. propagation loss; however, some components are undoubtedly attributable to equipment, processing, and incremental representation of continuous functions. The influence of navigation error would require considerable investigation if data were available. In the absence of such data, however, we observe that in few instances would shifting single run traces a few miles one way or another make them match significantly better.

3. The statistical tests applied in Section III-B (particularly the tests for equality of means) assume that each sample is from a normal population and that samples are independent. The normalcy of data distribution was accepted, but not proven, by testing distributions. Because the model was distinctly biased the existence of model error is not seriously questioned; but if the correspondence between model and data had been better, deeper investigation of data distribution would be necessary. The same caution applies to the sample independence. Sample spaces are one mile apart. Considering the five minute averages and the 18 knot projector velocity, we find some overlap from sample to sample, but not enough to question the results.

4. We observed earlier that reducing the model error to zero still leaves a significant experimental error for the user of model predictions to contend with. This is true, however, only so long as the experimental error remains the same. Since the experimental error is probably due largely to variation of uncontrolled variables, the obvious means of reducing it, conceptually at least, is to "control" one or more of the now uncontrolled variables. The recent introduction of bottom topography data into the model was an attempt to do this. We will not conjecture here what variables might be controlled, either experimentally or in the model, but merely point out that the sampling error limitation prevails only for the particular set of controlled variables now used.

5. Finally, perhaps the model should not be expected to predict the true mean loss at a point in space. Rather, a model might be configured to predict the general behavior of the loss function over an interval. This could conceptually be more useful to the user who is interested in intensity integrals over an interval.



CUMULATIVE DISTRIBUTION OF ERRORS

FIGURE 25

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V SUMMARY OF RESULTS

When a propagation loss prediction, for a certain range, is compared to a measurement made at the same range, the difference between the two is a combination of both error in the model and experimental error in the measurement. With the model runs and loss measurements used here, the combined errors were such that 50% of measurements differed by 5.5 db or less from the model and 95% differed by 13 db or less. It was found that agreement between the model and the measurements was significantly better at close range (out to about 40 miles) than at long ranges (100 miles and over) where the model consistently predicted too little loss.

Propagation loss measurements were made at source-receiver ranges of 0 to 115 miles, and the measurements were repeated four times over this interval. It was found that successive measurements at the same points were approximately normally distributed in db and log normal in intensity ratio. The standard deviation of the error distribution did not vary significantly with range, and was calculated to be 3.5 db.

The model error is considered to be the difference between the loss value predicted by the model, for a certain range, and the conceptual "true" average loss at the same range. The model error can be conveniently described in terms of two components; i.e. bias error and tracking error. Bias error describes the average offset of the model over some range interval and tracking error describes the fluctuation of the model about the conceptual "true" value. In the data used here, the bias error was found to vary from near zero at close range (0 to 40) miles to about -9 db at ranges from 75 to 115 miles. The tracking error can be described as an approximately normally distributed random variable with a standard deviation of 2.45 db and a mean of zero.

The sea bottom under the experimental area sloped gently from one end to the other; measurements were made both in the up slope and down slope directions. Differences in the two resulting loss curves were generally obscured by the presence of significant experimental error, and the curves differed little. The model predicted roughly equivalent losses in each direction, and it could not be shown that use of the local bottom topography by the model improved the predictions.

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VI RECOMMENDATIONS

In view of the substantial experimental error found here, it is recommended that future propagation loss measurement experiments be designed to provide adequate data concerning experimental error.

At sea experiments should be conducted to investigate the mechanisms contributing to the experimental error.

The variation of experimental error over a wide area and over substantial time periods should be investigated.

Similar comparisons should be made between model predictions and loss measurements conducted in other areas with different parameters. Comparisons should be made, for example, in an area where convergence zone propagation is evident.

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APPENDIX A

TABULATED PROPAGATION LOSS DATA

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Appendix A contains a tabulation of measurement data, model predictions and calculations arranged by sample. The results associated with each buoy are listed on separate pages. A single line of figures constitutes the data pertaining to one sample space.

The significance of the column headings is as follows:

COLUMN	
1 (R)	The range between the projector and the sonobuoy for measurement data. The horizontal range between source and receiver for model data.
2,3,4,5	The column headings are run identification numbers for the VASSEL XV measurements. Each of the columns contains propagation loss measurements, in db, for a single projector pass.
6 ($\overline{P/L}$)	Average propagation loss. The arithmetic average of the individual measurements in that sample.
7 (MP)	Propagation loss predicted by the model for that sample.
8 (S_i^2)	Variance of measurements in that sample.
9 (ε)	Deviation, in db, between sample mean and model prediction.
10 (SS)	Sample size (number of measurements in the sample).
11 (T-10)	Indicates whether model value was accepted (0) or rejected (R) as the true mean of that sample at the 10% level of significance
12 (T-1)	Indicates whether model value was accepted (0) or rejected (R) as the true mean of that sample at the 1% level of significance.

APPENDIX A

BUOY C

R	28D1	28D4	29D1	30D6	$\overline{P/L}$	MP	S_i^2	δ	SS	T-10	T-1
						63.3					
2		61.9	65.5	67.6	65.0	73.7	8.5	+8.7	3	R	R
3		73.1	71.8	77.5	74.1	85.3	8.9	+10.8	3	R	R
4		82.1	78.1	80.0	80.1	85.4	4.0	+5.3	3	R	R
5		78.5	78.4	77.0	78.0	85.2	1.6	+7.2	3	R	R
6		78.0	77.7	77.9	77.9	85.1	.7	+7.2	3	R	R
7		83.3	80.7	81.4	81.8	85.0	1.8	+3.2	3	R	O
8		80.6	81.4	78.4	79.8	84.8	2.2	+5.0	3	R	R
9		78.5	79.9	77.3	78.6	84.7	.9	+6.1	3	R	R
10		81.2	79.2	78.2	79.5	84.6	2.4	+5.1	3	R	R
11		84.9	80.4	83.6	83.0	84.5	5.4	+1.5	3	O	O
12			82.6	82.4	82.5	84.3	---	+1.8	2	O	O
13			78.3	77.7	78.0	84.2	---	+6.2	2	R	R
14	78.6	79.6	78.5	77.4	78.5	84.1	.8	+5.6	4	R	R
15	79.4	82.0	81.5	78.0	80.2	83.9	3.4	+3.7	4	R	O
16	81.4	84.0	84.7	78.4	82.6	83.7	8.2	+1.1	4	O	O
17	88.7	84.7	88.4		87.3	83.6	5.3	-3.7	3	R	O
18	88.5	84.6			86.5	83.5	---	-3.6	2	O	O
19	85.7	86.1			85.9	83.4	---	-2.5	2	O	O
20	86.3	86.1			86.2	83.6	---	-2.6	2	O	O
21	84.4	87.2	87.4		86.3	83.8	2.8	-2.5	3	O	O
22	79.6	84.8	83.7		82.7	83.9	7.5	-1.2	3	O	O
23	77.1	80.6	80.6		79.4	84.0	4.1	+4.6	3	R	O
24	77.5	78.8	79.9		78.7	84.1	1.4	+5.4	3	R	R
25	79.5	78.3	80.9		79.6	84.2	1.7	+4.6	3	R	O
26	82.4	78.8	82.8		81.3	84.4	4.9	+3.1	3	O	O
27	84.2	82.3	86.3		84.3	84.5	4.0	+ .2	3	O	O
28	80.9	85.5	83.6	80.0	82.5	84.6	6.2	+2.1	4	O	O
29	79.0	82.1	81.3	83.6	81.5	84.8	2.3	+3.3	4	R	O
30		81.1	81.4	87.0	83.2	85.0	11.0	+1.8	3	O	O
31		83.4	82.6	83.5	83.2	85.1	.3	+1.9	3	O	O
32		89.8	85.0	82.8	85.9	85.1	12.8	- .8	3	O	O
33	89.1	93.6	89.7	84.0	89.1	85.9	15.5	-3.2	4	R	O
34	89.2	94.5	92.9	86.3	90.7	86.3	13.6	-4.3	4	R	R
35	89.6		91.1	89.5	90.1	89.7	.9	- .4	3	O	O
36	90.6		89.9	91.3	90.6	86.4	.5	-4.2	3	R	O
37			91.5	91.4	91.5	86.7	---	-4.8	2	R	O
38			95.0	90.5	92.7	86.8	---	-5.9	2	R	O
39			94.4	89.1	91.7	87.7	---	-4.0	2	R	O
40				88.4	88.4	88.0	---	- .4	1	O	O

APPENDIX A

BUOY F

R	28D1	28D4	29D1	30D6	$\overline{P/L}$	MP	S_i^2	δ	SS	T-10	T-1
1						61.4	---				
2						69.1	---				
3						75.9	---				
4						85.0	---				
5	83.5			86.5	85.0	84.8	---	- .2	2	0	0
6	85.3			80.3	82.8	84.7	---	+1.9	2	0	0
7	97.2			82.0	84.6	84.6	---	0	2	0	0
8	86.2	86.2	81.6	83.3	84.4	84.4	6.3	0	4	0	0
9	83.8	83.2	86.6	83.5	84.3	84.3	2.5	0	4	0	0
10	82.5	80.7	88.6	85.3	84.3	82.5	11.9	-1.8	4	0	0
11	79.3	82.4	82.6	84.5	82.2	84.1	4.6	+1.9	4	0	0
12	78.2	82.8	78.9	85.1	81.2	83.9	10.7	+2.7	4	0	0
13	89.8	86.3	79.4	85.9	83.1	83.8	12.3	+ .7	4	0	0
14	86.7	89.5	81.6	86.9	86.2	83.5	11.0	-2.7	4	0	0
15	91.0	91.8	83.1	87.4	88.3	83.4	15.8	-4.9	4	R	R
16	87.1	87.4	84.0	87.5	86.5	83.2	2.6	-3.3	1	R	0
17	86.6	88.2	86.8	86.2	86.9	83.1	.8	-3.8	4	R	0
18	87.4	86.8	86.4	87.1	86.9	82.9	.2	-4.0	4	R	0
19	84.0	85.6	85.2	90.0	86.2	82.9	6.9	-3.3	4	R	0
20	79.0	82.6	83.9	83.5	82.2	83.0	5.0	+ .8	4	0	0
21	78.3	81.0	81.6	82.8	80.9	83.2	4.0	+2.1	4	0	0
22	79.7	80.2	80.5	85.9	81.6	83.2	8.4	+1.6	4	0	0
23		84.0	80.9	85.9	83.6	83.3	6.4X	- .3	3	0	0
24		85.7	84.1	85.1	85.0	83.3	.6X	-1.7	3	0	0
25			80.9		80.9	83.4	---	+2.5	1	0	0
26			79.1		79.1	83.4	---	+5.7	1	R	0
27	88.9			88.6	88.8	83.4	---	-5.4	2	R	0
28	93.2	89.9		86.2	89.8	83.5	11.6X	-6.3	3	R	R
29	87.1	96.6		86.2	90.0	83.6	33.2X	-6.4	3	R	R
30	86.6	98.3		87.2	90.7	83.7	42.5X	-7.0	3	R	R
31	90.7	96.4		93.2	93.4	84.0	8.2X	-8.6	3	R	R
32	92.8	91.8		92.1	91.6	84.4	.5X	-7.2	3	R	R
33	93.2	88.6	91.7	91.8	91.3	84.3	3.8	-7.0	4	R	R
34	93.2	92.3	91.1	94.2	92.7	85.4	1.8	-7.3	4	R	R
35	91.7	97.1	92.8	96.3	96.0	86.6	10.0	-10.0	4	R	R
36	87.7	96.6	93.4	96.1	93.4	87.3	19.1	-6.1	4	R	R
37	87.0	91.8	93.7	97.4	92.2	86.5	23.6	-5.7	4	R	R
38	86.4	92.8	90.0	97.0	91.6	85.5	18.7	-6.1	4	R	R
39	88.0	.	88.0	94.3	90.1	87.3	13.2X	-2.8	3	0	0
40	87.9		89.5	92.8	90.1	88.9	6.3X	-1.2	3	0	0

APPENDIX A

BUOY B

R	28D1	28D4	29D1	30D6	$\overline{P/L}$	MP	S_i^2	δ	SS	T-10	T-1
35											
36		91.3			91.3	83.3	---	-8.0	1	R	R
37		90.3			90.3	85.3	---	-5.0	1	O	O
38		90.5		92.6	91.6	87.5	---	-4.1	2	R	O
39		94.0		92.4	93.2	89.4	---	-3.8	2	R	O
40		95.1		92.9	94.0	89.4	---	-4.6	2	R	O
41		93.8	94.3	93.8	94.0	89.4	.5	-4.6	3	R	O
42		94.2	90.6	96.0	93.7	89.4	7.9	-3.3	3	R	O
43		93.4	89.6	94.3	92.4	89.4	6.3	-3.0	3	O	O
44		90.7	88.5	90.2	89.8	89.3	1.4	- .5	3	O	O
45		87.6	88.6	89.1	88.4	90.3	.6	+1.9	3	O	O
46			89.9	86.9	88.4	91.5	---	+2.9	2	O	O
47			92.1	85.8	88.9	91.0	---	+2.1	2	O	O
48			94.8	84.9	89.9	89.1	---	- .8	2	O	O
49			95.6	84.4	90.0	89.0	---	-1.0	2	O	O
50			94.7	86.4	90.6	89.0	---	-1.6	2	O	O
51			95.4	87.2	91.3	89.0	---	-2.3	2	O	O
52		107.4	96.9	88.2	97.5	89.1	92.5	-8.4	3	R	R
53	90.0	103.9	95.7		96.5	89.2	48.9	-7.3	3	R	R
54	90.0	101.7	93.0		94.9	88.6	36.9	-6.3	3	R	R
55	89.3	99.6			94.5	89.2	---	-5.3	2	R	O
56	87.0	96.8			91.9	89.3	---	-2.6	2	O	O
57	85.4	95.4			90.4	89.4	---	-1.0	2	O	O
58	85.8	95.5	93.1		91.5	89.5	17.0	-2.0	3	O	O
59	87.6	95.7	92.9		92.1	89.4	17.0	-2.7	3	O	O
60	89.6	96.1	93.1		92.9	89.7	10.6	-3.2	3	R	O
61	89.6	98.5	94.7		94.3	91.2	20.0	-3.1	3	R	O
62	90.2	99.9	96.7	88.2	93.8	92.6	30.0	-1.2	4	O	O
63	89.9	100.6	98.6	89.6	94.7	92.5	33.0	-2.2	4	O	O
64	88.7	101.8	96.2	90.5	94.3	92.4	35.6	-1.9	4	O	O
65	88.9	101.0	93.5	90.8	93.6	89.7	28.2	-3.9	4	R	O
66	89.4	98.4	94.1	91.3	93.3	89.0	15.3	-4.3	4	R	R
67	88.6	98.3	98.0	91.6	94.1	89.8	23.1	-4.3	4	R	R
68	89.3	98.5	100.5	90.8	94.8	90.2	32.5	-4.6	4	R	R
69			101.5	91.3	96.4	92.2	---	-4.2	2	R	O
70			95.8	93.2	94.5	95.0	---	+ .5	2	O	O
71	89.8		93.5	90.9	91.4	94.7	3.7	+3.3	3	R	O
72	87.1		92.9	90.6	90.2	90.9	8.6	+ .7	3	O	O
73	87.4		93.1	88.7	89.7	91.1	9.0	+1.4	3	O	O
74	88.4		92.4	86.1	89.0	91.1	10.2	+2.1	3	O	O
75			92.1	85.9	89.0	91.7	---	+2.7	2	O	O

APPENDIX A

BUOY G

R	28D1	28D4	29D1	30D6	$\overline{P/L}$	MP	S_i^2	δ	SS	T-10	T-1
35											
36											
37											
38											
39	89.0				89.0	89.3	-0-	+ .3	1	O	O
40	93.8				93.8	89.4	-0-	-4.4	1	O	O
41	93.7			93.6	93.7	89.5	---	-4.2	2	R	O
42	94.2			92.8	93.5	89.5	---	-4.0	2	R	O
43	95.2		90.8	92.0	92.7	89.5	5.2	-3.2	3	R	O
44	96.1	95.1	88.8	90.3	92.6	89.5	12.8	-3.1	4	R	O
45	95.2	95.5	91.2	90.6	93.1	89.5	5.8	-3.6	4	R	O
46	94.1	97.1	94.8	90.3	94.1	89.6	8.0	-4.5	4	R	R
47	92.0	100.2	95.9	90.2	94.4	89.5	19.6	-4.9	4	R	R
48	91.3	97.1	98.3	92.6	94.8	89.0	11.6	-5.8	4	R	R
49	92.7	97.2	95.3	94.7	95.0	89.5	14.4	-5.5	4	R	R
50	93.6	98.3	95.7	97.0	96.2	89.4	12.1	-6.8	4	R	R
51	94.7	95.8	100.2	96.6	96.8	89.4	5.7	-7.2	4	R	R
52	94.1	94.7	101.4	94.7	96.2	89.4	11.6	-6.8	4	R	R
53	94.0	95.3	99.3	94.3	95.7	89.4	6.0	-6.3	4	R	R
54	96.5	96.3	98.3	96.4	96.9	89.3	1.5	-7.6	4	R	R
55	94.1	95.4	98.8	94.1	95.6	89.2	4.9	-6.4	4	R	R
56	92.6	95.3	98.6	93.6	95.0	89.4	7.0	-5.6	4	R	R
57		93.2	98.3	95.7	95.7	89.4	6.3	-6.3	3	R	R
58		91.5	99.8	95.5	95.6	89.5	17.2	-6.1	3	R	R
59		90.2	101.6	95.0	95.6	89.7	17.8	-5.9	3	R	R
60		90.4	98.4	92.4	93.7	89.3	17.4	-4.4	3	R	O
61	88.1	92.2	96.8	88.1	91.3	90.0	17.2	-1.3	4	O	O
62	94.0		97.2		95.6	92.7	---	-2.9	2	O	O
63	98.6				96.6	92.6	-0-	-6.0	1	R	O
64	99.2			88.6	93.9	92.6	---	-1.3	2	O	O
65	99.7			90.4	95.1	92.5	---	-2.6	2	O	O
66	98.9	102.2		96.2	99.1	92.4	9.0	-6.7	3	R	R
67	97.4	101.5		96.3	98.4	92.3	7.5	-6.1	3	R	R
68	97.4	100.8		96.7	98.3	92.4	4.6	-5.9	3	R	R
69	96.8	102.0	99.6	98.2	99.2	92.3	4.9	-6.9	4	R	R
70	96.5	102.9	100.9	98.2	99.6	92.4	8.1	-7.2	4	R	R
71	99.0	101.3	98.9	98.3	99.4	92.0	1.8	-7.4	4	R	R
72	101.4	99.8	97.8	99.8	99.8	91.2	1.9	-8.6	4	R	R
73	103.5	99.7	99.0	98.1	100.1	91.9	5.7	-8.2	4	R	R
74		99.2	100.2	99.1	99.5	92.1	.4	-7.4	3	R	R
75		101.0	100.3	102.2	101.2	92.1	.9	-9.1	3	R	R
76		104.0	101.8	99.5	101.8	93.0	10.9	-8.8	3	R	R

APPENDIX A

BUOY A

R	28D1	28H4	29D1	30H6	$\overline{P/L}$	MP	S_i^2	δ	SS	T-10	T-1
75			97.7	102.6	100.2	92.7	---	-7.5	2	R	R
76			97.7	101.9	99.8	92.7	---	-7.1	2	R	R
77			97.5	100.3	98.9	92.7	---	-6.2	2	R	O
78			98.6	100.6	99.6	92.8	---	-6.8	2	R	R
79			99.9	101.6	100.8	92.7	---	-8.1	2	R	R
80		107.4	99.2	101.3	102.6	92.3	18.2	-10.3	3	R	R
81		106.0	99.7		102.9	92.8	---	-10.1	2	R	R
82		101.8	102.5		102.1	92.9	---	-9.2	2	R	R
83		101.3			101.3	93.3	-0-	-8.0	1	R	O
84		99.4			99.4	95.4	-0-	-4.0	1	O	O
85		97.1			97.1	92.0	-0-	-5.1	1	O	O
86		98.4	104.2		101.3	92.3	---	-9.0	2	R	R
87		99.1	100.6		99.9	92.8	---	-7.1	2	R	R
88			98.6		98.6	92.8	-0-	-5.8	1	R	O
89			98.0		98.0	92.8	-0-	-5.2	1	O	O
90			100.2	95.2	97.7	92.4	---	-5.3	2	R	O
91		99.9		94.8	97.4	92.2	---	-5.2	2	R	O
92	100.3	99.2		95.7	98.4	92.3	5.8	-6.1	3	R	R
93	99.1	99.6		96.3	98.3	94.7	3.2	-3.6	3	R	O
94	99.2	99.7		97.1	98.7	94.6	2.0	-4.1	3	R	O
95	98.4	95.6	100.6	95.8	97.6	87.3	7.1	-10.3	4	R	R
96	98.0	96.9	101.4	96.7	98.3	88.1	4.8	-10.2	4	R	R
97	97.9	96.7	101.9	97.8	98.8	87.4	5.3	-11.4	4	R	R
98	94.4	96.3	99.8	99.4	97.5	89.0	6.7	-8.5	4	R	R
99	92.3	95.8	99.9	104.8	98.2	89.9	27.6	-8.3	4	R	R
100	93.3	98.5	99.8	106.6	99.6	90.5	30.0	-9.1	4	R	R
101	98.1	103.8	100.6	107.3	102.2	90.7	12.9	-11.5	4	R	R
102	100.9	102.1	105.0	104.3	103.3	92.3	3.7	-11.0	4	R	R
103	103.8	101.5	107.0	105.1	104.4	90.3	5.4	-14.1	4	R	R
104	109.0	102.7	107.6	102.9	105.6	90.6	9.8	-15.0	4	R	R
105		101.8	108.0	98.2	102.3	94.6	25.2	-8.7	3	R	R
106		102.7	104.6	95.5	100.9	95.8	23.1	-5.1	3	R	O
107		103.3	103.0	103.3	103.2	98.7	.1	-4.5	3	R	O
108		101.3	104.0		102.7	93.3	---	-9.4	2	R	R
109		100.0			100.0	93.7	-0-	-6.3	1	R	O
110						93.9	-0-				
111	105.1				105.1	96.6	-0-	-8.5	1	R	O
112	104.3			100.1	102.2	95.7	---	-6.5	2	R	R
113	104.1				104.1	96.5	-0-	-7.5	1	R	O

APPENDIX A

BUOY H

R	2SP1	2SP4	2SD1	3SD6	$\overline{P/L}$	MP	S_i^2	δ	SS	T-10	T-1
75				110.9	110.2	91.0	-0-	-19.9	1	R	R
76				111.2	111.2	91.0	-0-	-20.2	1	R	R
77						91.0			0		
78				104.4	104.4	91.0	-0-	-15.4	1	R	R
79	107.5			96.8	102.2	91.0	---	-11.2	2	R	R
80	106.6			94.5	100.6	91.0	---	-9.6	2	R	R
81	104.7			92.9	98.8	91.0		-7.8	2	R	R
82	103.2		108.1	92.9	101.4	91.1	60.2	-10.3	3	R	R
83	102.5		104.8	99.5	107.3	91.3	7.1	-11.0	3	R	R
84	97.5		104.2	107.0	102.9	91.5	23.9	-11.4	3	R	R
85	98.5		101.4	111.2	103.7	91.7	44.3	-12.0	3	R	R
86	104.1	102.0	102.5	101.3	102.6	91.9	1.1	-10.7	4	R	R
87	99.7	99.6		97.4	98.7	93.7	1.4	-5.0	3	R	O
88	96.3	97.2		100.3	97.9	93.6	4.2	-4.3	3	R	O
89	93.1	97.2		97.0	95.8	93.8	5.4	-2.0	3	O	O
90	95.9	98.2		91.5	95.2	92.6	11.6	-2.6	3	O	O
91	104.3	99.1		89.8	97.7	93.6	55.6	-4.1	3	R	O
92	107.1	96.0	102.3		101.3	93.6	36.8	-8.2	3	R	R
93	104.0	95.9	101.5		100.5	93.5	17.3	-7.0	3	R	R
94	99.0	94.2	103.1		99.4	93.5	29.9	-5.9	3	R	R
95	97.9	93.7	105.7	102.5	100.0	93.3	27.6	-6.5	4	R	R
96	99.3	97.5	103.6	102.3	100.7	93.4	7.9	-7.3	4	R	R
97	101.5	98.1	103.7	105.1	102.1	93.4	9.3	-8.7	4	R	R
98		99.9	105.2	102.8	103.6	93.4	10.6	-10.2	3	R	R
99		104.2	105.0	104.1	104.4	93.3	.3	-11.1	3	R	R
100		105.6	100.0	105.0	103.5	93.1	9.5	-10.4	3	R	R
101		105.6	98.3	107.0	103.8	93.6	19.2	-10.8	3	R	R
102	101.4	105.4		104.6	103.8	93.2	4.5	-10.6	3	R	R
103	99.6			105.4	102.5	93.2	---	-9.3	2	R	R
104	97.8			105.6	101.7	93.0	---	-8.7	2	R	R
105	97.3			106.4	101.9	93.0	---	-8.9	2	R	R
106	98.3	102.6		97.5	99.5	92.8	7.5	-6.7	3	R	R
107	100.7	101.7		92.5	98.3	92.7	25.5	-5.6	3	R	R
108	101.1	102.6		94.2	99.3	92.9	20.1	-6.4	3	R	R
109	99.4	103.3		102.0	102.2	93.4	8.7	-9.2	3	R	R
110	99.1	105.0	107.2	105.0	104.1	93.0	12.1	-11.1	4	R	R
111	102.6	104.3	107.4	102.8	104.3	92.6	5.0	-11.7	4	R	R
112	106.5	106.7	108.0	100.2	105.4	92.9	12.2	-12.5	4	R	R
113		105.8		97.4	101.6	92.9	---	-8.7	2	R	R
114	109.5	105.2		95.0	103.2	93.0	55.9	-10.2	3	R	R
115	105.0	102.1		94.0	100.4	93.3	32.6	-4.9	3	R	O

(REVERSE SIDE BLANK)